

November 29, 2017

Brent J. Fields
Secretary
U.S. Securities and Exchange Commission
100 F Street, NE
Washington DC 20549-1090

RE: Notice of Designation of a Longer Period for Commission Action on a Proposed Rule Change to Adopt a New NYSE Arca Equities Rule 8.900 and to List and Trade Shares of the Royce Pennsylvania ETF, Royce Premier ETF, and Royce Total Return ETF under Proposed NYSE Arca Equities Rule 8.900 (Release No. 34-81977; File No. SR-NYSEArca-2017-36)

Dear Mr. Fields,

On behalf of Blue Tractor Group, LLC ("Blue Tractor") I am again pleased to provide the U.S. Securities and Exchange Commission (the "Commission") with comments regarding the Commission's October 30, 2017 notice (the "Notice") designating a longer period whether to approve or disapprove the rule change application submitted on April 14, 2017 by NYSE Arca, Inc. (the "Exchange"). 1, 2

This comment letter complements the two letters I submitted to the Commission on July 18, 2017 and October 31, 2017. <sup>3</sup> Please note that this letter contains an updated statistical report from Dr. Anthony Hayter from the University of Denver – see attached Exhibit One.

Fundamental to my comments is the intellectual property developed by Precidian Investments LLC ("Precidian") that underpin the three exchange traded funds (the "ETF Funds") sub-advised by Royce & Associates, LP ("Royce") that the Exchange proposes to list and trade. <sup>4, 5, 6</sup>

As importantly, this letter references the memorandum published by the Commission's Division of Economic Risk and Analysis on November 16, 2017 entitled "Inferring Non-Transparent ETF Portfolio Holdings" ("DERA" and the "DERA Memorandum"). <sup>7</sup> Of note, the DERA Memorandum validates that the Precidian ETF structure can be reverse engineered by a variety of statistical and optimization techniques.

It is a fact that reverse engineering a non-transparent portfolio is an exercise in statistics and data analysis. Whether it is or is not possible to reverse engineer the Precidian ETF structure is a quantitative issue that

<sup>&</sup>lt;sup>1</sup> See https://www.sec.gov/rules/sro/nysearca/2017/34-81977.pdf (Release No. 34-81977; File No. SR-NYSEArca-2017-36)

<sup>&</sup>lt;sup>2</sup> As background, I am the founder of Blue Tractor Group, LLC, which on July 31, 2017 filed a third amended and restated application for exemptive relief with the Commission for the Shielded Alpha<sup>™</sup> ETF structure. I am a graduate of the University of London (mathematics) in England and have worked and consulted for over 30 years in both England and United States for many financial institutions, primarily developing and constructing quantitative models related to alpha generation and risk management. I am the sole inventor of the methods and ideas underpinning the Shielded Alpha<sup>™</sup> ETF structure, which is a completely different concept to the non-transparent exchange traded fund structures proposed by Precidian and others that are currently being reviewed by the Commission.

<sup>&</sup>lt;sup>3</sup> See https://www.sec.gov/comments/sr-nysearca-2017-36/nysearca201736.htm (see the two letters from Terence W. Norman)

<sup>&</sup>lt;sup>4</sup> See <a href="https://www.nyse.com/publicdocs/nyse/markets/nyse-arca/rule-filings/filings/2017/NYSEArca-2017-36,%20Re-file.pdf">https://www.nyse.com/publicdocs/nyse/markets/nyse-arca/rule-filings/filings/2017/NYSEArca-2017-36,%20Re-file.pdf</a> (Release No. 30-80553; File No. SR-NYSEArca-2017-36)

<sup>&</sup>lt;sup>5</sup> https://www.roycefunds.com/

<sup>&</sup>lt;sup>6</sup> https://www.sec.gov/Archives/edgar/data/1701878/000114420417018966/v463050\_n1a.htm

https://www.sec.gov/comments/sr-nysearca-2017-36/nysearca201736.htm#studies (see SEC Staff Studies and Reports)



cannot be sugar coated by the qualitative protests by Messrs. McCabe and Criscitello in their letters to the Commission <sup>8</sup> - mathematics is the only arbiter to prove or disprove the issue.

On that note, it remains perplexing that Precidian or its consultants haven't to date presented additional quantitative evidence to counter the claims made in my July 18, 2017 and October 31, 2017 letters.

If Precidian still firmly believe it is 'highly unlikely' that their structure can be reverse engineered then they must present detailed mathematical evidence to counter my specific examples and they should not be allowed to generalize simply by referencing the discredited studies prepared by Drs. Cooper and Glosten.

No doubt Precidian has again privately approached the Commission in an attempt to assuage concerns about reverse engineering and resulting predatory front running. While Precidian has demonstrated little interest in public discourse, I would respectfully suggest that any new statistical evidence they present be released by the Commission into the public domain so that it can be peer reviewed to confirm or refute any claims contained within.

Peer review by Blue Tractor and DERA of the studies prepared by Drs. Cooper and Glosten provided a vital counter balance to the now wholly discredited claim and belief of Precidian that their ETF structure cannot be reverse engineered.

# So, to summarize:

- 1. Precidian continues to rely on the analyses prepared by Drs. Cooper and Glosten that are now discredited;
- 2. To date Precidian is unable to present mathematical evidence that refutes the concerns raised in my two previous letters (and accompany reports by Dr. Hayter);
- 3. Precidian says that the concerns raised by Blue Tractor emanate from a 'financial conflict of interest' and 'an attempt to stifle innovation' rather than recognize that the mathematics they have relied on are fundamentally flawed;
- 4. Since Precidian's core argument on reverse engineering has been shown to be false, then what confidence can the Commission have in the veracity of other major claims they have made, including predatory front running, efficient markets, arbitrage, hedging and the like (see public comment letters listing these issues from Simon Goulet and Gary Gastineau <sup>9</sup>); and
- 5. Furthermore, DERA has now provided independent confirmatory analysis that the Precidian ETF structure can be reverse engineered (see "DERA Memorandum" section).

<sup>8</sup> See https://www.sec.gov/comments/sr-nysearca-2017-36/nysearca201736.htm (see letters from Messrs. Criscitello and McCabe)

<sup>9</sup> See https://www.sec.gov/comments/sr-nysearca-2017-36/nysearca201736.htm (see letters from Simon Goulet & Gary Gastineau)



#### **DERA Memorandum**

The November 16, 2017 DERA Memorandum fully supports the concerns I document in my July 18, 2017 and October 31, 2017 letters that Precidian's 'stylised methodology' for the verified intra-day indicative value ("VIIV") is highly vulnerable to reverse engineering. <sup>10</sup>

DERA's most telling observations were on pages 3 through 6 where they:

- 1. Question Dr. Cooper and Dr. Glosten's formulation of the reverse engineering problem;
- 2. Question the statistical methods employed by Dr. Cooper and Dr. Glosten; and
- 3. Question Dr. Cooper and Dr. Glosten's conclusions.

In stark contrast the DERA Memorandum states on page 6 that, "Hayter estimates the correct regression specification in levels..." and then concludes with the statement, "...the two-pass aspect of his regression approach is an example of an alternative estimation technique that potentially improves upon the standard OLS and LASSO estimations..."

The DERA Memorandum then presents a clever approach that poses an ordinary least squares problem as a quadratic program ("QP-OLS").

The bottom line is that in contrast to Drs. Cooper and Glosten, both DERA and Dr. Hayter have demonstrated how to correctly formulate the problem of reverse engineering the Precidian ETF structure and then present methods to do so.

Additional key observations in the DERA Memorandum include:

- 1. By limiting the number of stocks in a fund and the fund's investable universe, reverse engineering is made easier;
- 2. The success of reverse engineering is dependent upon **both** correct formulation of the regression problem and the method used;
- 3. The success of an attempt to reverse engineer is dependent upon the number of observations available:
  - a. For a small stock universe, a single day's observations may be sufficient;
  - b. For a larger stock universe, multi-day observation may be required; and
  - c. As the expected number of fund holdings increase in both small and larger stock universes, while the problem of reverse engineering becomes more complex it does not become intractable:
- 4. Affirmation that just because one party is unable to undertake reverse engineering it **does not follow** that another party will be unable to do so; and

<sup>&</sup>lt;sup>10</sup> I note that the DERA Memorandum refers only to the analysis from Dr. Hayter contained in my July 18, 2017 letter.



5. That initial successful iterations in a reverse engineering problem should be considered the lower bound of what is achievable. Indeed, DERA states on page 6 of their memorandum that, "Any success in reverse-engineering the portfolio weights should therefore be considered a lower bound [emphasis added] on what is possible".

The DERA Memorandum puts to rest any doubt that Precidian's 'stylized methodology' for VIIV cannot be reverse engineered. The claim by Messrs. McCabe and Criscitello that their structure is immune to reverse engineering is patently false.

To that end, an active mutual fund manager considering using the Precidian ETF structure is by default, considering utilizing a transparent active ETF structure.

However, insofar that it will be 'transparent' to predatory traders, it will not trade efficiently for investors like currently approved transparent actively managed ETFs do because it's use of a third-party structure to facilitate creation, arbitrage and hedging means it will be less efficient in both the primary and secondary markets. In that light, better for the active mutual fund manager to simply opt for today's transparent active ETF structures already granted exemptive relief by the Commission.

Finally, from the work undertaken by DERA and Dr. Hayter, a basic 'recipe' for reverse engineering is now in the public domain:

- 1. Determine the size of the stock universe;
- Overestimate the number of stocks expected in the fund (readily available from quarterly disclosures);
- 3. Determine if single or multi-day observations are needed;
- 4. Mathematically formulate the problem correctly;
- 5. Incorporate additional information gleaned from the fund's SAI or other public sources into the math e.g. fund turnover %, sector constraints etc.;
- 6. Run "artificial simulations' to fine tune the method using real data from the fund's previously disclosed portfolio information; and
- 7. Once the portfolio is "cracked", monitor over many trading days to detect the fund manager's strategy and habits and when actual trades occur.

I would also note that if an ETF fund using the Precidian structure also has a 'mirror' mutual fund then both funds are at risk of predatory front running.

Please refer to attached Exhibit One of this letter for an in-depth review of the DERA Memorandum by Dr. Hayter, along with a further improved method for reverse engineering.



### **Front Running**

The traders with the quantitative skill set to reverse engineer are of course the same traders who will then bring their talents to predatory front running.

Who are they? They are the traders and strategists that work for hedge funds, proprietary trading desks and specialty trading houses, both domestically and off-shore. These traders combine capital markets savvy with graduate and doctorate level degrees in computer sciences, physics and mathematics. <sup>11</sup> These firms usually have unlimited computer resources at their disposal and their compensation models motivate traders to take risk and be aggressive.

The techniques outlined in the DERA Memorandum and by Dr. Hayter are trivial to these mathematically gifted individuals.

So once the portfolio has been "cracked" these traders will then develop techniques to take maximum advantage of the fund's trading patterns and strategy. Time is on the predatory traders' side – once the portfolio becomes visible they can sit back and fine tune their strategies before entering the market unobserved by the fund. They do not need to trade every day in front of the fund; they will do so only when profitable.

The DERA Memorandum and the work by Dr. Hayter clearly demonstrate that no fund using the Precidian ETF structure, regardless of the size of the fund or investable universe, can be certain that their undisclosed portfolio hasn't been subject to reverse engineering and that 'alpha' is being skimmed by a predatory trader.

I say this with tongue in cheek but a prospectus for funds using the Precidian ETF structure would have to include under the risk section language to the effect that, "These funds are extremely vulnerable to predatory front-running which could negatively impact investor returns".

# **Conclusion**

Precidian lists in Mr. Criscitello's October 11, 2017 letter a long list of potential investor benefits under a non-transparent ETF structure. <sup>12</sup> Ironically, what they don't document is evidence that investors want a retrogressive non-transparent investment structure. If the benefits Mr. Criscitello lists can be accrued in a much more transparent structure, why would anyone want to invest in a non-transparent 'black box'? Indeed, the success of the ETF market is built on the foundation of **transparency**. Why would approval of a wholly opaque, non-transparent ETF be innovative?

Blue Tractor would always encourage the Commission to permit novel products that improve investor choice, fulfill a real product demand, enhance market efficiency and capital formation, overcome known financial obstacles through real innovation, while always maintaining the spirit and integrity of existing federal securities laws that promote 'fair markets' and provide investor protection. This last condition is inviolable.

<sup>11</sup> https://www.nytimes.com/2016/02/23/business/dealbook/a-new-breed-of-trader-on-wall-street-coders-with-a-phd.html (Note: this article is for reference only and no explicit or implicit suggestion is made that any of the firms or individuals profiled are assumed to undertake predatory trading activities)

<sup>&</sup>lt;sup>12</sup> Note that Mr. Criscitello cannot claim that the Precidian ETF structure would accrue all these benefits since it can be readily reverse engineered and will have inefficient primary and secondary markets as described in letters Simon Goulet and others submitted to the Commission.



Some market observers maintain that the Commission should approve every application it receives for novel ETF structures and 'should stand back and let the market decide' winners and losers. Interesting thought and so long as an application for a novel structure clearly meets the legal spirit of federal securities law to promote both 'fair markets' and investor protection, that could indeed be a viable approach.

However to recap, the Precidian ETF structure does not solve any investor demand for non-transparent products, it will do nothing to enhance market efficiency, it does not overcome any financial obstacles since it is highly vulnerable to reverse engineering (and therefore front-running) and the asymmetric disclosure of confidential portfolio information to privileged third parties clearly runs a foul of existing federal securities law. Therefore, the Precidian ETF structure cannot be viewed with the same lens as novel product applications that do meet all of these conditions.

Moreover, there is now confirmatory evidence from DERA that the Precidian ETF structure can be reverse engineered.

For these reasons, the Exchange's rule change application should be disapproved.

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Thank you in advance for your consideration of my commentary. I welcome any questions the Commission may have as a result and can be reached at the commentary.

Sincerely,

Terence W. Norman Founder Blue Tractor Group, LLC



# HayterStatistics.com

November 26th, 2017

Notes on the November 16<sup>th</sup>, 2017, Memorandum from the Division of Economic and Risk Analysis, entitled

"Inferring Non-Transparent ETF Portfolio Holdings"

### 1. Overview.

These notes provide some comments on the November 16th, 2017, Memorandum from the Division of Economic and Risk Analysis (DERA) of the U.S. Securities and Exchange Commission, entitled "Inferring Non-Transparent ETF Portfolio Holdings".

The DERA Memorandum provides a generally perceptive and well thoughtout discussion of the problem of reverse engineering a portfolio. The DERA Memorandum considers the analyses presented in several reports on this subject, and also presents some original simulation results of their own. The DERA Memorandum comes across as being a fair and unbiased assessment of the situation.

It is interesting to note that in the DERA Memorandum the problem is specified in exactly the same way that I specified it in my initial report "The Reverse Engineering of Portfolio Compositions", July 17<sup>th</sup>, 2017. The equation in the middle of page 1 of the DERA Memorandum is exactly identical to equation (1) on page 9 of my report, with exactly the same weights which sum to one.

The importance of setting up the problem correctly in the first place cannot be overstated. This is essential to the development of an algorithm that will successfully achieve reverse engineering. It should be pointed out that neither Ricky Cooper nor Lawrence Glosten provided this equation in their work, and they did not demonstrate that they had specified the problem correctly.



Obviously, a failed attempt at reverse engineering using a misspecified approach does not provide any illumination on the prospects of successfully achieving reverse engineering using a properly specified approach.

# 2. REO Analysis.

The REO approach is interesting in that it is based upon returns from one time increment to the next (using 10 second increments in this case), rather than being based on price levels. However, as the DERA Memorandum explains, this does not allow the precise estimation of what we are actually looking for. Consequently, this approach adds unnecessary difficulty and noise to the problem.

I agree with the DERA Memorandum that "Over multiple days, weight estimates based on returns will under-perform estimates based on price levels."

# 3. Ricky Cooper's Analysis.

I agree with the discussion in the DERA Memorandum concerning how the problem is set up. In the Cooper analysis the problem is poorly set up, so that it is estimating the wrong thing, with weights that do not sum to one.

I agree with the DERA Memorandum that this analysis and interpretation has "not fully accounting for the effects of scaling", and I agree with the DERA Memorandum that there are inherent difficulties with regressing on unscaled prices (as in the Cooper analysis) rather than on scaled prices, so that there are flaws in the operation and interpretation of the Cooper analysis.

### 4. Lawrence Glosten's Analysis.

I agree with the DERA Memorandum that "for the purpose of reverse engineering an ETF's portfolio holdings, small prediction errors are not the primary objective" and that with respect to LASSO "it is not obvious a priori that it should outperform OLS." In fact, I think that there are clearly limitations to taking any "off-the-shelf" package and applying it to the problem of reverse engineering a portfolio. Much more success is attainable with the development of an algorithm specifically for the reverse engineering problem at hand.



Again, I agree with the discussion in the DERA Memorandum concerning how the problem is set up. Again, as the DERA Memorandum explains, "Because the regressors are not scaled, the regression produces estimates of the number of shares of each security held." These are quantities which do not sum to one.

I think that the DERA Memorandum makes an important point in stating that "Therefore, it is possible that other LASSO implementations could achieve more accurate estimates of portfolio holdings." This is a recognition that any failed attempt at reverse engineering does not prove that all attempts must fail.

The DERA Memorandum confirms this by also stating that "Finally, Glosten's finding that OLS and LASSO are unable to accurately recover portfolio holdings for a larger universe of available portfolio securities (the Russell 1000) does not rule out the possibility that other techniques might perform better on stock universes of the same size."

# 5. Blue Tractor's Analysis.

This section of the DERA Memorandum comments on my initial report "The Reverse Engineering of Portfolio Compositions", July 17<sup>th</sup>, 2017. However, I issued a supplemental report on October 19<sup>th</sup>, 2017, which is not referred to in the DERA Memorandum.

The supplemental report considered more challenging scenarios than were considered in the initial report, and it provided further analyses to confirm that the reverse engineering of a portfolio is achievable with a substantial degree of accuracy.

Specifically, the differences between the analyses contained in the supplemental report and in my initial report are:

- Whereas the initial report only considered reporting of prices at time points 15-seconds apart, which provides 1,560 time points throughout a complete trading day, in the supplemental report the reporting of prices at time points 1-second apart is considered, which provides 23,400 time points throughout a complete trading day.
- Whereas the initial report considered a universe of k = 100 potential stocks, in the supplemental report the more challenging scenario of a universe of k = 1,000 potential stocks is considered. The reverse engineering of



- portfolios consisting of both 130 and 80 stocks out of this universe of k = 1,000 potential stocks is demonstrated.
- Finally, the implementation of more sophisticated multi-day reverse engineering methodologies is demonstrated in the supplemental report, which more closely model how a serious attempt at reverse engineering would be made in practice.

While it is correct that my analysis "uses a single 'average' correlation value across the simulated stock universe" as pointed out in the DERA Memorandum, I actually present worst-case, average, and best-case scenarios for the stock variabilities and correlations in an attempt to encompass real scenarios.

I note the recognition in the DERA Memorandum ("Hayter estimates the correct regression specification") of the importance of setting up the problem correctly.

As the DERA Memorandum notes, "Hayter uses a two-pass approach: the above regression is estimated, any stocks with statistically insignificant coefficients are removed from the universe, and the regression is re-estimated" and "Nonetheless, the two-pass aspect of his regression approach is an example of an alternative estimation technique that potentially improves upon the standard OLS and LASSO estimations above." I think this illustrates the advantages that can accrue from an algorithm designed specifically for the reverse engineering problem at hand.

In fact, the second stage of my approach is more of an optimization approach (constrained quadratic minimization problem) similar to the technique mentioned on page 8 of the DERA Memorandum ("posing the least-squares problem as a quadratic program"). Again, rather than taking any "off-the-shelf" package and applying it to the problem of reverse engineering a portfolio, this illustrates that more success is attainable with the development of an algorithm specifically for the reverse engineering problem at hand.

As the DERA Memorandum notes, "Hayter shows that with 5 days of data, he can reverse engineer portfolios in this small universe to a very high degree of accuracy: his weights are only off by an average of 0.000776 relative to the true portfolio weight of 0.025, exhibit very few false-positives, and never exclude a stock that is in the true portfolio." This demonstrated successful reverse



engineering in this scenario with what I would consider a relatively simple and naïve algorithm.

The DERA Memorandum also notes "With only one day of data and 15 second VIIV increments, the results are less accurate, but this is much less data than the one second increments that Precidian's ETF structure would provide" and "On a larger universe with 1 second data, the results would probably lie somewhere in between these two extremes." I agree with these comments if the relatively simple and naïve algorithm used in my initial report were adopted. However, the more sophisticated multi-day reverse engineering methodologies illustrated in my supplemental report demonstrated successful reverse engineering for these more challenging scenarios.

I think it is also important to stress that, as pointed out in my reports, in practice it would be expected that experts with knowledge of the specific stocks involved, with some prior historical information about the portfolio, and with an understanding of prevailing market conditions, for example, would be able to fine-tune any methodology in order to substantially improve its performance.

#### 6. DERA Simulations.

The DERA Memorandum contains an important observation: "Any success in reverse-engineering the portfolio weights should therefore be considered a lower bound on what is possible." Similarly, if an attempt at reverse engineering is made which turns out to be unsuccessful, then this in no way shows that reverse engineering cannot be done. It should not be forgotten that all of the analyses of reverse engineering that have been performed so far only provide a "lower bound" on what a determined approach at reverse engineering could achieve.

The DERA simulations provide further useful insights into the reverse engineering problem, and since like REO they are using returns, which is not the best specification, and which adds difficulty and noise to the problem, it is clear that their results could be improved upon.

I agree with the findings of the DERA simulations that:

"the effectiveness of reverse engineering decreases as the size of the universe increases"



- "funds that invest in a larger fraction of the universe are harder to reverse engineer"
- "algorithms perform better with more observations relative to the universe size".

Finally, I like the ingenuity and insight that "DERA was able to improve estimation results using the fact that each error term  $\epsilon_t$  is a result of rounding to the nearest penny, so  $\$-0.005 < \epsilon_t < \$0.005$  for all t" so that they find benefit by "posing the least-squares problem as a quadratic program". This illustrates the fact that an ingenious and determined protagonist will be able to fine-tune any methodology in order to substantially improve its performance.

### 7. New Results.

Tables 1.1-2.3 present new simulation results that demonstrate a new improved algorithm for conducting reverse engineering. The tables presented here are updated versions of Tables 1.1-2.3 contained within my supplemental report of October 19<sup>th</sup>, 2017.

A portfolio of 130 stocks out of a potential universe of 1,000 stocks is considered, with a scaled portfolio price of 50 (Tables 1.1-1.3) and 30 (Tables 2.1-2.3), and with the reporting of prices at time points 1-second apart. In all cases, in addition to the scaling of the portfolio price, "shielding" of the portfolio is achieved by rounding the portfolio price to r = 2 decimal places (in other words, to the nearest penny). Furthermore, as discussed in my supplemental report, three scenarios of the daily stock volatilities  $\sigma_d$  and correlations  $\rho$  are considered.

The results presented here use an improved version of the sophisticated multi-day reverse engineering methodology illustrated in my supplemental report. Specifically, similarly to the approach discussed and adopted in the DERA simulations, a quadratic programming minimization algorithm with constraints was implemented, which makes full use of the fact that the weights are positive and sum to one, and which can also make use of the known bounds on the errors as pointed out by DERA.

I have used metrics which I think most clearly demonstrate the effectiveness of the reverse engineering, namely:

Number of stocks incorrectly excluded from the estimated portfolio.



- Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.
- Number of stocks incorrectly included in the estimated portfolio.
- Sum of estimated weights for stocks incorrectly included in the estimated portfolio.

For completeness, I have also included the standard Pearson correlation coefficient and the Spearman rank correlation coefficient between the true weights and the estimated weights.

The results presented in Tables 1.1-2.3 clearly show the success of the reverse engineering, and they also have a substantial improvement over the results presented in my supplemental report. This illustrates the important point that there is always the potential to provide a new insight that can lead to the improvement of a reverse engineering methodology simply by formulating the algorithm slightly differently.

Anthony Hayter, Ph.D. HayterStatistics.com November 26<sup>th</sup>, 2017



Table 1.1									
Universe of k = 1,000 potential stocks. Portfolio initially contains 130 stocks with equal weights 1/130.									
	Initial scaled value of a portfolio share = 50. Portfolio share price rounded to r=2 decimal places.								
	1-second reporting (23,400 values per day).								
	Worst-case scenario for daily volatility and correlations: $\sigma_d$ = 0.0137 and $\rho$ = 0.551.								
	Table entries are average values based on M = 10 simulations.								
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.	Pearson correlation between true weights and estimated weights.	Spearman rank correlation between true weights and estimated weights.			
	Portfolio weights unchanged.								
Day 1	1.9	0.0015	4.0	0.0165	0.96	0.98			
Day 2	0.0	0.0010	0.1	0.0003	0.98	1.00			
Day 3	0.0	0.0009	0.0	0.0000	0.99	1.00			
Day 4	0.0	0.0008	0.0	0.0000	0.99	1.00			
Day 5	0.0	0.0007	0.0	0.0000	0.99	1.00			
Day 6	0.0	0.0006	0.0	0.0000	0.99	1.00			
Day 7	0.0	0.0005	0.0	0.0000	1.00	1.00			
Day 8	0.0	0.0005	0.0	0.0000	1.00	1.00			
Day 9	0.0	0.0005	0.0	0.0000	1.00	1.00			
Day 10	0.0	0.0005	0.0	0.0000	1.00	1.00			
		Portfoli	o weights change da	y to day.					
Day 1	1.0	0.0015	3.3	0.0138	0.98	0.96			
Day 2	0.0	0.0010	0.2	0.0007	1.00	0.98			
Day 3	0.0	0.0008	0.0	0.0000	1.00	0.99			
Day 4	0.0	0.0007	0.0	0.0000	1.00	0.99			
Day 5	0.0	0.0007	0.0	0.0000	1.00	0.99			
Day 6	0.0	0.0006	0.0	0.0000	1.00	0.99			
Day 7	0.0	0.0006	0.0	0.0000	1.00	0.99			
Day 8	0.0	0.0006	0.0	0.0000	1.00	1.00			
Day 9	0.0	0.0005	0.0	0.0000	1.00	1.00			
Day 10	0.0	0.0005	0.0	0.0000	1.00	1.00			



Table 1.2										
Universe of k = 1,000 potential stocks. Portfolio initially contains 130 stocks with equal weights 1/130.										
	Initial scaled value of a portfolio share = 50. Portfolio share price rounded to r=2 decimal places.									
	1-second reporting (23,400 values per day).									
	Average scenario for daily volatility and correlations: $\sigma_d$ = 0.0173 and $\rho$ = 0.278.									
Table entries are average values based on M = 10 simulations.										
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.	Pearson correlation between true weights and estimated weights.	Spearman rank correlation between true weights and estimated weights.				
	Portfolio weights unchanged.									
Day 1	0	0.0009	0	0.0000	0.99	1.00				
Day 2	0	0.0006	0	0.0000	0.99	1.00				
Day 3	0	0.0005	0	0.0000	1.00	1.00				
Day 4	0	0.0004	0	0.0000	1.00	1.00				
Day 5	0	0.0004	0	0.0000	1.00	1.00				
Day 6	0	0.0004	0	0.0000	1.00	1.00				
Day 7	0	0.0003	0	0.0000	1.00	1.00				
Day 8	0	0.0003	0	0.0000	1.00	1.00				
Day 9	0	0.0003	0	0.0000	1.00	1.00				
Day 10	0	0.0003	0	0.0000	1.00	1.00				
		Portfoli	o weights change da	y to day.						
Day 1	0	0.0009	0	0.0000	0.99	1.00				
Day 2	0	0.0006	0	0.0000	0.99	1.00				
Day 3	0	0.0005	0	0.0000	1.00	1.00				
Day 4	0	0.0005	0	0.0000	1.00	1.00				
Day 5	0	0.0004	0	0.0000	1.00	1.00				
Day 6	0	0.0004	0	0.0000	1.00	1.00				
Day 7	0	0.0004	0	0.0000	1.00	1.00				
Day 8	0	0.0004	0	0.0000	1.00	1.00				
Day 9	0	0.0004	0	0.0000	1.00	1.00				
Day 10	0	0.0004	0	0.0000	1.00	1.00				



# Table 1.3

Universe of k = 1,000 potential stocks. Portfolio initially contains 130 stocks with equal weights 1/130.

Initial scaled value of a portfolio share = 50. Portfolio share price rounded to r=2 decimal places.

1-second reporting (23,400 values per day).

Best-case scenario for daily volatility and correlations:  $\sigma_d$  = 0.0237 and  $\rho$  = 0.181.

### Table entries are average values based on M = 10 simulations.

	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.	Pearson correlation between true weights and estimated weights.	Spearman rank correlation between true weights and estimated weights.
		Port	folio weights unchar	nged.		
Day 1	0	0.0006	0	0.0000	1.00	1.00
Day 2	0	0.0004	0	0.0000	1.00	1.00
Day 3	0	0.0003	0	0.0000	1.00	1.00
Day 4	0	0.0003	0	0.0000	1.00	1.00
Day 5	0	0.0003	0	0.0000	1.00	1.00
Day 6	0	0.0002	0	0.0000	1.00	1.00
Day 7	0	0.0002	0	0.0000	1.00	1.00
Day 8	0	0.0002	0	0.0000	1.00	1.00
Day 9	0	0.0002	0	0.0000	1.00	1.00
Day 10	0	0.0002	0	0.0000	1.00	1.00
		Portfoli	o weights change da	y to day.		
Day 1	0	0.0006	0	0.0000	1.00	1.00
Day 2	0	0.0004	0	0.0000	1.00	1.00
Day 3	0	0.0003	0	0.0000	1.00	1.00
Day 4	0	0.0003	0	0.0000	1.00	1.00
Day 5	0	0.0003	0	0.0000	1.00	1.00
Day 6	0	0.0003	0	0.0000	1.00	1.00
Day 7	0	0.0003	0	0.0000	1.00	1.00
Day 8	0	0.0003	0	0.0000	1.00	1.00
Day 9	0	0.0003	0	0.0000	1.00	1.00
Day 10	0	0.0003	0	0.0000	1.00	1.00



			Table 2.1							
Universe of k = 1,000 potential stocks. Portfolio initially contains 130 stocks with equal weights 1/130.										
Initial scaled value of a portfolio share = 30. Portfolio share price rounded to r=2 decimal places.										
	1-second reporting (23,400 values per day).									
	Worst-case scenario for daily volatility and correlations: $\sigma_d$ = 0.0137 and $\rho$ = 0.551.									
Table entries are average values based on M = 10 simulations.										
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.	Pearson correlation between true weights and estimated weights.	Spearman rank correlation between true weights and estimated weights.				
		Port	folio weights unchar	nged.						
Day 1	36.4	0.0039	36.6	0.2052	0.71	0.69				
Day 2	19.9	0.0032	15.2	0.0701	0.83	0.85				
Day 3	8.3	0.0023	9.2	0.0387	0.91	0.93				
Day 4	5.7	0.0020	4.3	0.0163	0.93	0.96				
Day 5	5.4	0.0019	0.9	0.0032	0.94	0.97				
Day 6	0.8	0.0015	0.7	0.0026	0.96	0.99				
Day 7	1.1	0.0014	0.2	0.0007	0.97	0.99				
Day 8	2.1	0.0014	0.0	0.0000	0.97	0.99				
Day 9	0.3	0.0013	0.1	0.0003	0.98	1.00				
Day 10	0.1	0.0012	0.0	0.0000	0.98	1.00				
		Portfoli	o weights change da	y to day.						
Day 1	35.3	0.0039	35.6	0.1993	0.71	0.70				
Day 2	19.2	0.0032	11.8	0.0548	0.84	0.87				
Day 3	7.7	0.0022	9.5	0.0398	0.91	0.93				
Day 4	4.1	0.0019	3.4	0.0141	0.94	0.97				
Day 5	5.0	0.0019	1.5	0.0056	0.94	0.97				
Day 6	1.2	0.0015	1.8	0.0067	0.96	0.98				
Day 7	1.3	0.0015	0.5	0.0018	0.97	0.99				
Day 8	2.1	0.0015	0.2	0.0006	0.97	0.99				
Day 9	0.2	0.0013	0.3	0.0010	0.98	0.99				
Day 10	0.2	0.0012	0.1	0.0003	0.98	0.99				



Table 2.2										
Universe of k = 1,000 potential stocks. Portfolio initially contains 130 stocks with equal weights 1/130.										
Initial scaled value of a portfolio share = 30. Portfolio share price rounded to r=2 decimal places.										
	1-second reporting (23,400 values per day).									
	Average scenario for daily volatility and correlations: $\sigma_d$ = 0.0173 and $ ho$ = 0.278.									
	Table entries are average values based on M = 10 simulations.									
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.	Pearson correlation between true weights and estimated weights.	Spearman rank correlation between true weights and estimated weights.				
		Port	folio weights unchar	nged.						
Day 1	13.9	0.0026	13.1	0.0629	0.87	0.89				
Day 2	3.4	0.0017	2.7	0.0100	0.95	0.97				
Day 3	1.0	0.0014	0.6	0.0022	0.97	0.99				
Day 4	0.1	0.0012	0.1	0.0003	0.98	1.00				
Day 5	0.0	0.0010	0.0	0.0000	0.98	1.00				
Day 6	0.0	0.0009	0.0	0.0000	0.99	1.00				
Day 7	0.0	0.0009	0.0	0.0000	0.99	1.00				
Day 8	0.0	0.0008	0.0	0.0000	0.99	1.00				
Day 9	0.0	0.0008	0.0	0.0000	0.99	1.00				
Day 10	0.0	0.0007	0.0	0.0000	0.99	1.00				
		Portfoli	o weights change da	y to day.						
Day 1	14.0	0.0025	12.8	0.0636	0.87	0.89				
Day 2	1.8	0.0017	3.4	0.0132	0.95	0.98				
Day 3	0.6	0.0014	0.9	0.0032	0.97	0.99				
Day 4	0.0	0.0012	0.2	0.0007	0.98	1.00				
Day 5	0.0	0.0010	0.0	0.0000	0.98	1.00				
Day 6	0.0	0.0010	0.0	0.0000	0.99	1.00				
Day 7	0.0	0.0009	0.0	0.0000	0.99	1.00				
Day 8	0.0	0.0008	0.0	0.0000	0.99	1.00				
Day 9	0.0	0.0008	0.0	0.0000	0.99	1.00				
Day 10	0.0	0.0008	0.0	0.0000	0.99	1.00				



Table 2.3										
Universe of k = 1,000 potential stocks. Portfolio initially contains 130 stocks with equal weights 1/130.										
	Initial scaled value of a portfolio share = 30. Portfolio share price rounded to r=2 decimal places.									
	1-second reporting (23,400 values per day). Best-case scenario for daily volatility and correlations: $\sigma_d$ = 0.0237 and $ ho$ = 0.181.									
Table entries are average values based on M = 10 simulations.										
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.	Pearson correlation between true weights and estimated weights.	Spearman rank correlation between true weights and estimated weights.				
		Port	folio weights unchar	nged.						
Day 1	1.0	0.0014	2.4	0.0097	0.97	0.98				
Day 2	0.0	0.0009	0.1	0.0003	0.99	1.00				
Day 3	0.0	0.0008	0.0	0.0000	0.99	1.00				
Day 4	0.0	0.0007	0.0	0.0000	0.99	1.00				
Day 5	0.0	0.0006	0.0	0.0000	0.99	1.00				
Day 6	0.0	0.0006	0.0	0.0000	1.00	1.00				
Day 7	0.0	0.0005	0.0	0.0000	1.00	1.00				
Day 8	0.0	0.0005	0.0	0.0000	1.00	1.00				
Day 9	0.0	0.0005	0.0	0.0000	1.00	1.00				
Day 10	0.0	0.0004	0.0	0.0000	1.00	1.00				
		Portfoli	o weights change da	y to day.						
Day 1	0.7	0.0014	1.7	0.0070	0.97	0.99				
Day 2	0.0	0.0010	0.1	0.0003	0.99	1.00				
Day 3	0.0	0.0008	0.0	0.0000	0.99	1.00				
Day 4	0.0	0.0007	0.0	0.0000	0.99	1.00				
Day 5	0.0	0.0006	0.0	0.0000	0.99	1.00				
Day 6	0.0	0.0006	0.0	0.0000	0.99	1.00				
Day 7	0.0	0.0006	0.0	0.0000	1.00	1.00				
Day 8	0.0	0.0005	0.0	0.0000	1.00	1.00				
Day 9	0.0	0.0005	0.0	0.0000	1.00	1.00				
Day 10	0.0	0.0005	0.0	0.0000	1.00	1.00				