

October 31, 2017

Brent J. Fields
Secretary
U.S. Securities and Exchange Commission
100 F Street, NE
Washington DC 20549-1090

RE: Notice of Designation of a Longer Period for Commission Action on a Proposed Rule Change to Adopt a New NYSE Arca Equities Rule 8.900 and to List and Trade Shares of the Royce Pennsylvania ETF, Royce Premier ETF, and Royce Total Return ETF under Proposed NYSE Arca Equities Rule 8.900 (Release No. 34-80935; File No. SR-NYSEArca-2017-36)

Dear Mr. Fields,

I am writing the U.S. Securities and Exchange Commission (the "Commission") with regard to the Commission's July 31, 2017 order notice (the "Order") instituting proceedings to approve or disapprove the rule change application submitted on April 14, 2017 by NYSE Arca, Inc. (the "Exchange").<sup>1, 2, 3</sup>

In addition to the proposed rule change, the Exchange wishes to list and trade the Royce Pennsylvania ETF, the Royce Premier ETF and the Royce Total Return ETF, non-transparent exchange traded funds (the "Funds")<sup>4</sup> which will operate using intellectual property developed by Precidian Investments LLC ("Precidian") and which are described in a Form N-1A Registration Statement and statement of Supplementary Additional Information ("SAI") filed on April 4, 2017 by Precidian ETF Trust II (the "Trust").<sup>5</sup>

The July 31, 2017 Order references four public comment letters pertaining to the Exchange's rule change application, including one submitted by me on July 18, 2017. My letter also included as Appendix One a detailed reverse engineering exercise prepared by Dr. Anthony Hayter from the University of Denver that presented mathematical proof that the conclusion reached by Precidian's consultant Dr. Ricky Cooper from the Illinois Institute of Technology, that it was "rather unlikely" that the Precidian ETF structure could be reverse engineered, was false.

<sup>&</sup>lt;sup>1</sup> See https://www.sec.gov/rules/sro/nysearca/2017/34-81267.pdf (Release No. 34-80935; File No. SR-NYSEArca-2017-36)

<sup>&</sup>lt;sup>2</sup> See <a href="https://www.nyse.com/publicdocs/nyse/markets/nyse-arca/rule-filings/filings/2017/NYSEArca-2017-36,%20Re-file.pdf">https://www.nyse.com/publicdocs/nyse/markets/nyse-arca/rule-filings/filings/2017/NYSEArca-2017-36,%20Re-file.pdf</a> (Release No. 30-80553; File No. SR-NYSEArca-2017-36)

<sup>&</sup>lt;sup>3</sup> As background, I am the founder of Blue Tractor Group, LLC, which on July 31, 2017 filed a third amended and restated application for exemptive relief with the Commission for the Shielded Alpha<sup>5M</sup> ETF structure. I am a graduate of the University of London (mathematics) in England and have worked and consulted for over 30 years in both England and United States for many financial institutions, primarily developing and constructing quantitative models related to alpha generation and risk management. I am the sole inventor of the methods and ideas underpinning the Shielded Alpha<sup>5M</sup> ETF structure, which is a completely different concept to the non-transparent exchange traded fund structures proposed by Precidian and others that are currently being reviewed by the Commission.

See https://www.sec.gov/Archives/edgar/data/1668791/000168035917000403/bluetractor40app7312017.htm (File No. 812-14625)

<sup>&</sup>lt;sup>4</sup> Precidian's proposed exchange traded fund structure is a non-transparent fund because no actual portfolio holdings are disclosed daily and the market will only know actual stock positions and their weightings in the fund on a quarterly basis.

<sup>&</sup>lt;sup>5</sup> See https://www.sec.gov/Archives/edgar/data/1701878/000114420417018966/v463050\_n1a.htm (File No. 811-23246)



On page twenty (20) of the Order the Commission summarizes my letter and notes that, "...The commenter refutes the Trust's statistical analysis that purports to demonstrate that the Funds' portfolio compositions could not be reverse engineered. The commenter's analysis concludes that reverse engineering of a Fund's portfolio is in fact "achievable with a substantial degree of accuracy.""

The Order concludes by asking interested parties for additional public comments or rebuttals to previously submitted comment letters by August 25, 2017 and September 8, 2017, respectively. Moreover, on page twenty-three (23) the Order notes, "Specifically, the Commission seeks comment on...the issues raised by the commenters..."

Because the proposed Funds will operate under an ETF structure that my July 18, 2017 letter mathematically demonstrates is at risk of reverse engineering by predatory parties, it was surprising no public rebuttal comments were received by September 8, 2017 from Precidian or its consultants that presented quantitative evidence refuting the statistics and analyses prepared by myself and Dr. Hayter.

Indeed, using the portfolio parameters utilized by Dr. Cooper and Precidian's "stylized methodology" for the verified intraday indicative value ("VIIV"), Dr. Hayter demonstrated many instances of being able to reverse engineer a portfolio. One questions why this has not been refuted by Precidian with any rigor.

Rebuttal letters the Commission eventually did receive from Precidian earlier this month containing some commentary refuting the reverse engineering concerns in my July 18, 2017 letter are of a general nature, offer no quantitative specifics and therefore should be discounted (received from Messrs. Mark Criscitello and Daniel McCabe on October 11 and 12, 2017, respectively).<sup>7</sup>

Mr. McCabe's rebuttal letter however does reference a recent analysis commissioned for Precidian by Dr. Lawrence Glosten of the Columbia Graduate School of Business (the "Glosten Case Study") and that is introduced as Exhibit F in Precidian's fourth amended and restated application for exemptive relief, filed September 29, 2017.<sup>8</sup>

Entitled "Analysis of the Ability to determine the Portfolio Underlying an Actively Managed ETF", the Glosten Case Study unsurprisingly concludes that the Precidian ETF structure cannot be reverse engineered with sufficient precision to allow for predatory front running.

As a result, I have prepared for public review this comment letter, including the attached Exhibit A, that refute the statistical methods and conclusions of the Glosten Case Study. **The three takeaways are:** 

- 1. Key statistical techniques employed in the examples used in the Glosten Case Study are of concern;
- 2. The conclusion that the Precidian ETF structure cannot be reverse engineered is false; and
- 3. The premise that a hedge portfolio can be ably constructed based upon a regression of the unknown portfolio price levels is false.

<sup>&</sup>lt;sup>6</sup> Precidian's 'stylized methodology' consists of: (1) Scaling the ETF to an initial value of \$20.00 and allowing it to range to no greater than \$60.00 before undertaking a stock split to bring it back down to \$20.00, (2) Calculating the VIIV using input prices that are the midpoint of the bid-ask for the portfolio constituents and (3) Truncating the value of the disseminated VIIV to two decimal places.

<sup>&</sup>lt;sup>7</sup> https://www.sec.gov/comments/sr-nysearca-2017-36/nysearca201736.htm (see letters from Messrs. Criscitello and McCabe)

<sup>8</sup> https://www.sec.gov/Archives/edgar/data/1396289/000114420417050779/v476219\_40appa.htm (File No. 812-14405)



Finally, I wish to reinforce with the Commission that the other concerns raised in my July 18, 2017 letter, in addition to reverse engineering, remain outstanding and have not to date been addressed by Precidian.

# **Glosten Case Study**

Mr. McCabe's October 12, 2017 letter specifically references the Glosten Case Study's three conclusions that, "...using a sophisticated regression technique, 1) analysts can do a little bit better than random guessing of the portfolio constituents; 2) cannot determine changes in weights as the estimated portfolio constituents change from day to day; and 3) nonetheless, the regression has a very tight fit allowing the construction of very good hedge portfolios".

It is disappointing that the Glosten Case Study examples as presented do not provide supporting statistical data and analysis, rather than sprinkles of statistical terminology and summary figures, especially since Mr. McCabe relies upon its conclusions in his letter to the Commission.

As a result, the reader must infer and 'read between the lines' at certain junctures. Nonetheless, enough can be gleaned from the work to call into question certain of Dr. Glosten's key statistical techniques and therefore resulting conclusions, as I will point out below.

#### Three Concerns with the Examples Presented in the Glosten Case Study

1. Dr. Glosten bases all his examples on a random portfolio of 130 names, a figure selected because it was," ...the number of names specified as an average in the Precidian Registration Statement".

I have reviewed the Precidian ETF Trust II preliminary N-1A registration statement and appended SAI filed on April 4, 2017 and find no such specification for the proposed ETF Funds that are to be subadvised by Royce & Associates, LP ("Royce")<sup>9</sup> and ClearBridge Investments, LLC ("ClearBridge")<sup>10</sup>. Unsurprisingly, both the preliminary N-1A and SAI are silent on the typical size of portfolio for each proposed Fund.

However, it's instructive to look at three small cap actively managed mutual funds currently issued by Royce and three mid, large and all-cap mutual funds from ClearBridge.

These six mutual funds appear to be the proxies for six of the proposed exchange traded funds because the ETF Funds:

- Will have identical portfolio managers; and
- Will have similar investment strategies (except to preclude foreign securities).

<sup>9</sup> https://www.roycefunds.com/

<sup>10</sup> https://www.clearbridge.com/



#### Royce and ClearBridge Funds

Sub-Advisor	Proposed ETF Fund	Current Mutual Fund	# Holdings in Mutual Fund **
Royce	Royce Pennsylvania ETF	Royce Pennsylvania Mutual Fund	329
	Royce Premier ETF	Royce Premier Mutual Fund	54
	Royce Total Return ETF	Royce Total Return Mutual Fund	274
ClearBridge <sup>11</sup>	ClearBridge Appreciation ETF	ClearBridge Appreciation Mutual Fund	79
	ClearBridge Large Cap ETF	ClearBridge Large Cap Growth Fund	49
	ClearBridge All Cap Value ETF	ClearBridge All Cap Value Fund	69

<sup>\*\*</sup> As at 9/30/17 for Royce Mutual Funds and 8/31/17 for ClearBridge Mutual Funds

Clearly there is wide dispersion in the number of holdings in the six mutual funds and while a simple average is a portfolio size of 142 securities, this is a specious figure since the fund strategies range from small to all-cap and the average is skewed by the large holdings in two of the Royce small cap funds.

Predatory traders will be eagerly looking at the much smaller portfolio sizes for four of these funds since the smaller the number of stocks within the fund the easier reverse engineering becomes.

2. Secondly, of significantly greater concern is that Dr. Glosten's examples employ stock (and ETF) **prices**, rather than stock (and ETF) **price returns**. The excerpt below is how Dr. Glosten describes the technique used for the examples presented in the Glosten Case Study:

The statistical technique for estimating the portfolio is determined by the following observations. If the investment universe is the Russell 1000, then the VIIV provided in second t is given by the following:

$$PU(t) = W_1P_1(t) + W_2P_2(t) + ... + W_{1000} P_{1000}(t),$$
  
 $P(t) = round(PU(t))$ 

where  $P_k(t)$  is the **quote midpoint of security k at second t** [emphasis added],  $W_k$  is the weight on security k determined from the portfolio the day before, and PU(t) is the **raw, unreported ETF price** [emphasis added] while P(t) is the **reported portfolio price, rounded to the penny** [emphasis added], at second t. Of course, most of the weights are zero.

As is commonly known, the volatility of a security (a.k.a. risk) is defined as the standard deviation of **price returns rather than actual prices**<sup>12</sup>.

This is of fundamental concern since use of prices, rather than price returns, can lead to spurious results as demonstrated below using a simple 10-stock illustration.

Based upon an annualised volatility of 10%, ten (10) time series of stock price returns (XRtn<sub>i</sub>) are generated from a normal distribution with mean 0.0 and per second volatility of  $\sigma_s$ , where  $\sigma_s$  is the per second equivalent of the annualised volatility of 10%.

<sup>&</sup>lt;sup>11</sup> Note: Although the Exchange has not proposed to list and trade the ClearBridge ETF Funds, inclusion of the selected ClearBridge mutual funds that are listed in the Trust's N-1A registration statement and appended SAI is instructive. Additionally, Bats BZX Exchange, Inc. proposes to list and trade the ClearBridge ETF Funds.

<sup>12</sup> https://en.wikipedia.org/wiki/Volatility (finance)



#### A correlation matrix is then prepared:

#### Randomly Generated Stock Price Returns Correlation Matrix (A)

XRtn <sub>i</sub>	1	2	3	4	5	6	7	8	9	10
1	1.000	0.007	-0.003	-0.007	-0.013	-0.003	0.004	-0.007	-0.004	-0.002
2	0.007	1.000	0.000	0.004	-0.004	-0.004	-0.004	-0.006	0.003	0.000
3	-0.003	0.000	1.000	-0.002	0.008	0.004	-0.002	0.010	-0.004	-0.013
4	-0.007	0.004	-0.002	1.000	0.007	-0.002	-0.002	0.003	-0.008	-0.003
5	-0.013	-0.004	0.008	0.007	1.000	-0.006	0.003	0.002	-0.006	-0.012
6	-0.003	-0.004	0.004	-0.002	-0.006	1.000	-0.005	-0.008	0.008	-0.008
7	0.004	-0.004	-0.002	-0.002	0.003	-0.005	1.000	-0.002	0.001	-0.001
8	-0.007	-0.006	0.010	0.003	0.002	-0.008	-0.002	1.000	-0.008	-0.004
9	-0.004	0.003	-0.004	-0.008	-0.006	0.008	0.001	-0.008	1.000	0.005
10	-0.002	0.000	-0.013	-0.003	-0.012	-0.008	-0.001	-0.004	0.005	1.000

As a comparison, let's instead observe the correlation matrix between ten (10) derived stock prices time series  $(X_i)$ , using the recursive relationship:

$$X_i(t+1) = X_i(t) * (1.0 + XRtn_i(t)) i = 1...10 and t = 1....23,400 and$$

where  $X_i(1)$  is a randomly drawn real number between 10.0 & 60.0.

#### Randomly Generated Stock Price Series Correlation Matrix (B)

Xi	1	2	3	4	5	6	7	8	9	10
1	1.000	0.779	-0.579	-0.294	0.189	0.066	-0.491	0.025	-0.665	0.154
2	0.779	1.000	-0.419	0.015	0.222	-0.263	-0.512	0.035	-0.415	0.218
3	-0.579	-0.419	1.000	0.013	-0.178	0.191	0.654	-0.282	0.314	-0.405
4	-0.294	0.015	0.013	1.000	0.011	-0.614	-0.380	0.273	0.429	0.307
5	0.189	0.222	-0.178	0.011	1.000	0.001	-0.100	-0.298	-0.370	-0.270
6	0.066	-0.263	0.191	-0.614	0.001	1.000	0.470	-0.281	-0.268	-0.477
7	-0.491	-0.512	0.654	-0.380	-0.100	0.470	1.000	-0.142	0.317	-0.744
8	0.025	0.035	-0.282	0.273	-0.298	-0.281	-0.142	1.000	0.355	0.145
9	-0.665	-0.415	0.314	0.429	-0.370	-0.268	0.317	0.355	1.000	-0.048
10	0.154	0.218	-0.405	0.307	-0.270	-0.477	-0.744	0.145	-0.048	1.000

As is observed, Matrix (B) based on stock price levels exhibits a high degree of positive and negative correlation between each of the series. In practical terms, an ETF market maker believing this to be correct would mistakenly put on an incorrect hedge. In other words, since all ten of the stock price time series  $(X_i)$  were generated using price returns that were uncorrelated, any correlations shown must be of spurious nature.

It is important to note that these tables clearly contradict the Glosten Case Study's third conclusion that the regression technique used allows, "...the construction of very good hedge portfolios".



For example, if  $X_1$  was the price of an ETF share and  $X_2,...X_{10}$  were the prices of individual stocks being used to hedge  $X_1$ , then a hedge portfolio comprising  $X_2,...X_{10}$  would yield totally different results as opposed to constructing a hedge portfolio for XRtn<sub>1</sub> using a portfolio comprising XRtn<sub>2</sub>,...,XRtn<sub>10</sub>.

A market maker would immediately observe from Matrix (A) that the ten (10) stock price return series are uncorrelated and a proper hedge portfolio for  $XRtn_1$  could then be assembled using other instruments. Whereas observation of Matrix B would suggest to a market maker a hedging portfolio for  $X_1$  could be constructed from  $X_2,...,X_{10}$ , when that would not be the case.

3. Thirdly, Dr. Glosten notes that, "Technically, the regression is "sparse" and statistical techniques have been developed to estimate such a model. The regression package Lasso is used."

In layperson terms, when there is a group of highly correlated variables, LASSO regression analysis tends to <u>arbitrarily</u> select only one when undertaking variable selection. Why is this important?

Because as illustrated in correlation Matrix (B) above, the use of price levels (rather than price returns as seen with correlation Matrix A) results in variables exhibiting a high degree of multicollinearity<sup>13</sup>, i.e. groups of highly correlated variables.

As a result, it is a widely recognized concern using LASSO that this regression technique "behaves erratically" in the present of multicollinear variables. Therefore, using LASSO will result in an <u>arbitrary selection process</u> to determine which stocks to include and which to exclude from the final model.

As a result, the rationale of using LASSO for the Glosten Case Study examples – a regression technique known to arbitrarily select variables - must be questioned, especially given what conclusions Precidian hoped to receive from Dr. Glosten.

I would also note that a predatory trader attempting to reverse engineer a portfolio would normally employ techniques that would allow them to 'manually filter out' unwanted variables, as well as use other available informational inputs, rather than blindly accepting the results of an automated statistical operation such as LASSO.

#### **Refuting the Glosten Case Study's Conclusions**

As noted, Mr. McCabe's October 12, 2017 letter repeats the conclusions from the Glosten Case Study that, "...using a sophisticated regression technique, 1) analysts can do a little bit better than random guessing of the portfolio constituents; 2) cannot determine changes in weights as the estimated portfolio constituents change from day to day; and 3) nonetheless, the regression has a very tight fit allowing the construction of very good hedge portfolios."

I will not directly address conclusion statements #1 and #2 above, except to say that they are false and would instead refer the reader to Dr. Hayter's initial reverse engineering paper (Appendix One) in my July

<sup>13</sup> https://en.wikipedia.org/wiki/Multicollinearity



18, 2017 comment letter and to Dr. Hayter's updated analysis dated October 19, 2017, attached as Exhibit A to this letter.

Both studies prepared by Dr. Hayter **conclusively demonstrate that Precidian's stylized methodology can be reverse engineered** with sufficient precision for predatory trading purposes (both portfolio holdings and weights).

To summarize, Dr. Hayter's initial study used portfolio parameters described by Dr. Ricky Cooper while his second study attached as Exhibit A utilizes portfolio parameters described by Dr. Glosten. As illustrated in Exhibit A, Dr. Hayter undertook a reverse engineering exercise using the portfolio parameters (130-stock portfolio/1,000-stock investable universe) that mimic the Glosten Case Study's examples and his results fully refute the hopeful conjecture of Dr. Glosten (and therefore Mr. McCabe and by inference, Dr. Cooper) that it is not possible to reverse engineer the Precidian actively managed ETF portfolio with sufficient precision.

### Indeed, Dr. Hayter summarizes on page nine (9) of attached Exhibit A that,

"It should be noted that reverse engineering, namely the determination of the composition of a portfolio based upon the prices of the individual potential stocks and a "shielded price" of the portfolio, is inherently a problem of statistics and data analysis. While it may be approached in a naïve manner through the implementation of only standard statistical regression techniques, the application of more sophisticated techniques and expertise is certain to provide more accurate and successful results."

#### Dr. Hayter then posits that,

"In fact, it is worthwhile to consider conceptually how it might be possible to establish that reverse engineering **cannot** be done (as has been claimed in the exhibits to Precidian's "Fourth Amended and Restated Application"), and on the other hand, conceptually how it might be possible to establish that reverse engineering **can** be done. Specifically, it is important to realize that if an attempt at reverse engineering is made which turns out to be unsuccessful, then this in no way shows that reverse engineering cannot be done. It merely shows that the particular methodology implemented in that attempt is not sufficient, and that the methodology employed is not based upon sufficiently sophisticated statistical techniques and expertise."

#### Dr. Hayter goes on to say that,

"Consequently, the claimed failures to achieve reverse engineering using the methodologies employed in the exhibits to Precidian's "Fourth Amended and Restated Application" (filed with the U.S. Securities and Exchange Commission on September 29, 2017) do not establish that reverse engineering cannot be done. The claimed failures simply show that those attempts at reverse engineering were too naïve and that they were insufficiently sophisticated."

#### And his final conclusion of the matter is that,

"On the other hand, the development of a methodology that can be shown to achieve the reverse engineering of a portfolio with a substantial degree of accuracy is sufficient to establish that reverse engineering can be done. The simulations presented in my initial report and in this report do exactly this."

Of course, Dr. Hayter does not employ the LASSO regression method used by Dr. Glosten. Note too that Dr. Hayter also carries out a reverse engineering analysis for an 80-stock portfolio derived from a 1,000-



stock investable universe as a further comparison exercise and to demonstrate what is achievable when the number of stocks within the portfolio decreases.

I will instead focus my remaining comments on conclusion #3 as the methods employed in the examples prepared by Dr. Glosten actually demonstrate that it is **not** possible to construct "... very good hedge portfolios".

Dr. Glosten observes that, "All but one of the regressions had R-Squareds that were above .95. The worst fit exhibited an R-Squared of .89. This analysis suggests that it is perfectly plausible that **a hedge portfolio** can be constructed [emphasis added] using the regression technology on the one-second prices, without any knowledge of the actual underlying portfolio."

Dr. Glosten therefore has concluded that the high value of the statistic R-Squared (" $R^2$ ")<sup>14</sup> is an indication that a hedge portfolio can be constructed. As a mathematician, I would immediately comment that students new to quantitative techniques quickly learn that  $R^2$  increases in value as more independent variables are added to a regression. It is therefore possible to construct a value of  $R^2$  as high as required simply by adding random variables. To therefore claim that a high value of  $R^2$  by itself is an indication that market makers can effectively hedge without knowledge of the actual underlying portfolio contents is naive.

As previously described, all the Glosten Case Study examples use stock price levels rather than stock price returns for regression and therefore Dr. Glosten's conclusion is suspect. Indeed, when employing stock price levels, a high value R<sup>2</sup> provides no assurances whatsoever of a well-defined model (even ignoring the recognised problems using LASSO regression analysis, as previously highlighted).

Dr. Glosten also observes that, "Four of the regressions had an R-Squared of 1 (to four decimal places) which means that only a trivial amount of variation in the ETF price is not explained by variation in the chosen equity prices. That is, statistically a portfolio can be constructed with a price that is very highly correlated with the ETF price. This is a familiar result as arbitragers have for long used a smaller portfolio to approximate the S&P 500 portfolio with very good results."

With all due respect, some observers could construe these statements to illustrate: (1) misunderstanding of how market makers construct hedge portfolios and (2) unfamiliarity with how arbitragers construct, "...a smaller portfolio to approximate the S&P 500 portfolio..."

I believe it instructive to demonstrate that a high value of R<sup>2</sup> does not mean a hedge portfolio can automatically be constructed:

Let Y represent a portfolio of unknown stocks.

Then form a time series of second by second prices (Y) for a single trade day.

Then calculate the 1-second price returns (YRtn) from Y.

Form two separate first order auto-regressions ("AR(1)") for both Y and YRtn.

<sup>&</sup>lt;sup>14</sup> https://en.wikipedia.org/wiki/Coefficient of determination



The two regressions will take the form:

Y= a + b \* Y{1} .....(1) 15 and 
$$YRtn = p + q * YRtn{1} .....(2)$$

and both (1) & (2) will be estimated by Ordinary Least Squares.

My own experience with quantitative methods tells me in advance that I will very likely obtain a very high value of R<sup>2</sup> using equation (1) because it is based upon stock price levels and price levels usually contain a trend. Thus, despite a high value of R<sup>2</sup> it is practically useless for hedging.

And as anticipated, estimation of equation (1) yields a R<sup>2</sup> value of 99.13%, which is consistent with the values reported by the Glosten Case Study. But its high value does not automatically imply that one can hedge the unknown portfolio that constitutes Y. After all, if Y is unknown then Y{1} must also be unknown, which means it cannot be used to hedge.

In stark contrast, estimation of equation (2) based upon stock price returns yields a R<sup>2</sup> value of only 6.67%; clearly a figure not conducive for constructing an efficient hedge portfolio.

As earlier described, the Glosten Case Study observed that, "All but one of the regressions had R-Squareds that were above .95. The worst fit exhibited an R-Squared of .89. This analysis suggests that it is perfectly plausible that **a hedge portfolio can be constructed** [emphasis added] using the regression technology on the one-second prices, without any knowledge of the actual underlying portfolio."

This statement as demonstrated, is misleading since the Glosten Case Study examples are based upon:

- 1. The use of stock price levels rather than stock price returns;
- 2. A single statistic i.e. R<sup>2</sup>; and
- 3. A statistical regression method (LASSO) that under certain well known and present conditions, <u>arbitrarily</u> selects variables.

Finally, Dr.Glosten states that, "...arbitragers have for long used a smaller portfolio to approximate the S&P 500 portfolio with very good results."

I would comment that unlike a non-transparent ETF where portfolio contents would be completely unknown, arbitragers are fully aware of the contents of the S&P 500 Index and can therefore ably manage both their statistical and non-statistical risk.

<sup>&</sup>lt;sup>15</sup> Note that {1} denotes a lag of 1 period (in this instance the lag is 1 second) and a, b, p and q are regression coefficients to be estimated by Ordinary Least Squares.



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Thank you in advance for your consideration of my commentary. I welcome any questions the Commission may have as a result and can be reached at

Sincerely,

Terence W. Norman Founder Blue Tractor Group, LLC



# **EXHIBIT A**

The Reverse Engineering October 19<sup>th</sup> of 2017 Portfolio Compositions Supplemental Report of Dr. Anthony Hayter HayterStatistics.com



#### Caveat

The opinions and results set forth in this report are based on an assessment of information currently available to its author. If, when, and to the extent that additional data and information are made available and can be properly evaluated, it is possible that the opinions and results set forth in this report will need to be supplemented and/or modified. The author reserves the right to do so if data and information later made available suggest any such supplementation and/or modification is appropriate.





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# Section I: The Reverse Engineering of Portfolio Compositions.

## Overview.

This is a supplemental report to my initial report "The Reverse Engineering of Portfolio Compositions" dated July 17<sup>th</sup>, 2017. This report considers more challenging scenarios than were considered in the initial report, and it provides further analyses to confirm that the reverse engineering of a portfolio is achievable with a substantial degree of accuracy.

Specifically, the differences between the analyses contained in this supplemental report and in my initial report are:

- Whereas the initial report only considered reporting of prices at time points 15-seconds apart, which provides 1,560 time points throughout a complete trading day, in this supplemental report the reporting of prices at time points 1-second apart is considered, which provides 23,400 time points throughout a complete trading day.
- Whereas the initial report considered a universe of k = 100 potential stocks, in this supplemental report the more





challenging scenario of a universe of k = 1,000 potential stocks is considered. The reverse engineering of portfolios consisting of both 130 and 80 stocks out of this universe of k = 1,000 potential stocks is demonstrated.

 Finally, the implementation of more sophisticated multi-day reverse engineering methodologies is demonstrated in this supplemental report, which more closely model how a serious attempt at reverse engineering would be made in practice.





#### □ Summary.

The simulations presented in this supplemental report again demonstrate that the reverse engineering of a portfolio is achievable with a substantial degree of accuracy. Moreover, this demonstration has been made for the more challenging scenario of a universe of k = 1,000 potential stocks.

As discussed in the initial report, the amount of "shielding" of the portfolio price through scaling and rounding of its value directly affects the accuracy of the reverse engineering, and also affects the time taken to obtain reasonable results. In addition, the information available from the 1-second and 15-second reporting of real stock prices rather than the simulated stock prices considered in this report may affect the accuracy of the reverse engineering, and the time taken to obtain reasonable results. However, for the realistic and challenging scenarios considered in this report it is clear that the reverse engineering of a portfolio is achievable with a substantial degree of accuracy.

The simulation results presented in this report demonstrate the implementation of a sophisticated multi-day reverse engineering methodology. However, in practice, it would be expected that experts with knowledge of the specific stocks involved, with some prior historical information about the portfolio, and with an understanding of prevailing market conditions, for example, would be able to fine tune a methodology such as this in order to substantially improve its performance.





Thus, the results presented in this report are intended to demonstrate the practicality of reverse engineering for these challenging scenarios, but they are not intended to demonstrate any optimal approach to the problem of reverse engineering nor to provide any optimal results. As explained, it would be expected that superior results could be obtained in practice by any serious attempt at reverse engineering.





#### Demonstrating Reverse Engineering.

As in the initial report, simulations have been conducted for this supplemental report in order to assess the accuracy that can be achieved by various reverse engineering methodologies for different scenarios.

It should be noted that reverse engineering, namely the determination of the composition of a portfolio based upon the prices of the individual potential stocks and a "shielded price" of the portfolio, is inherently a problem of statistics and data analysis. While it may be approached in a naïve manner through the implementation of only standard statistical regression techniques, the application of more sophisticated statistical techniques and expertise is certain to provide more accurate and successful results.

In fact, it is worthwhile to consider conceptually how it might be possible to establish that reverse engineering *cannot* be done (as has been claimed in the exhibits to Precidian's "Fourth Amended and Restated Application"), and on the other hand, conceptually how it might be possible to establish that reverse engineering *can* be done. Specifically, it is important to realize that if an attempt at reverse engineering is made which turns out to be unsuccessful, then this in no way shows that reverse engineering cannot be done. It merely shows that the particular methodology implemented in that attempt is not sufficient, and that the methodology employed is not based upon sufficiently sophisticated statistical techniques and expertise.





Consequently, the claimed failures to achieve reverse engineering using the methodologies employed in the exhibits to Precidian's "Fourth Amended and Restated Application" (filed with the U.S. Securities and Exchange Commission on September 29, 2017) do not establish that reverse engineering cannot be done. The claimed failures simply show that those attempts at reverse engineering were too naïve and that they were insufficiently sophisticated.

On the other hand, the development of a methodology that can be shown to achieve the reverse engineering of a portfolio with a substantial degree of accuracy is sufficient to establish that reverse engineering can be done. The simulations presented in my initial report and in this report do exactly this.





# ☐ Simulation Results.

The simulation results presented in this supplemental report consider eight scenarios:

- ➤ A portfolio of 130 stocks out of a potential universe of 1,000 stocks, with a scaled portfolio price of 50, and with the reporting of prices at time points 1-second apart (Tables 1.1-1.3).
- ➤ A portfolio of 130 stocks out of a potential universe of 1,000 stocks, with a scaled portfolio price of 30, and with the reporting of prices at time points 1-second apart (Tables 2.1-2.3).
- ➤ A portfolio of 80 stocks out of a potential universe of 1,000 stocks, with a scaled portfolio price of 50, and with the reporting of prices at time points 1-second apart (Tables 3.1-3.3).
- ➤ A portfolio of 80 stocks out of a potential universe of 1,000 stocks, with a scaled portfolio price of 30, and with the reporting of prices at time points 1-second apart (Tables 4.1-4.3).
- ➤ A portfolio of 130 stocks out of a potential universe of 1,000 stocks, with a scaled portfolio price of 50, and with the reporting of prices at time points 15-seconds apart (Tables 5.1-5.3).





- ➤ A portfolio of 130 stocks out of a potential universe of 1,000 stocks, with a scaled portfolio price of 30, and with the reporting of prices at time points 15-seconds apart (Tables 6.1-6.3).
- ➤ A portfolio of 80 stocks out of a potential universe of 1,000 stocks, with a scaled portfolio price of 50, and with the reporting of prices at time points 15-seconds apart (Tables 7.1-7.3).
- ➤ A portfolio of 80 stocks out of a potential universe of 1,000 stocks, with a scaled portfolio price of 30, and with the reporting of prices at time points 15-seconds apart (Tables 8.1-8.3).

In all cases, in addition to the scaling of the portfolio price, "shielding" of the portfolio is achieved by rounding the portfolio price to r = 2 decimal places. Furthermore, as discussed in the initial report, the following three scenarios of the daily stock volatilities  $\sigma_d$  and correlations  $\rho$  are considered:

- Worst-case scenario:  $\sigma_d = 0.0137$  and  $\rho = 0.551$ .
- $\triangleright$  Average scenario:  $\sigma_d = 0.0173$  and  $\rho = 0.278$ .
- **>** Best-case scenario:  $\sigma_d = 0.0237$  and  $\rho = 0.181$ .

The simulation results in this supplemental report show the implementation of a sophisticated multi-day reverse engineering methodology, which provides an indication of how a serious attempt at reverse engineering would be made in practice. Estimations of the portfolio are made at sequential time points (every day for the reporting of prices at time points 1-







second apart, and every three days for the reporting of prices at time points 15-seconds apart), and the methodology incorporates learning techniques so that the portfolio estimate at a particular point in time incorporates the knowledge gained from all of the previous estimates.

In each of the tables presented in this supplemental report, results are first shown for the situation in which the weights of the stocks in the portfolio remain unchanged. Results are then shown for the situation in which the weights of the stocks in the portfolio are allowed to change over the time period considered (for the reporting of prices at time points 1-second apart the weights are allowed to change at the end of each day, and for the reporting of prices at time points 15-seconds apart the weights are allowed to change at the end of every third day). The changes in the weights of the stocks in the portfolio are obtained by randomly selecting five of the stocks in the portfolio to have their weights increased by 10%, and by randomly selecting five of the stocks in the portfolio to have their weights decreased by 10%.

As expected, the tables presented in this supplemental report demonstrate that the reverse engineering is easier when there are only 80 instead of 130 stocks in the portfolio, when the portfolio is scaled to 50 rather than to 30, and when the weights of the stocks in the portfolio remain unchanged. Nevertheless, for the reporting of prices at time points 1-second apart, the most difficult scenario is presented in Table 2.1 (130 stocks, scaled price of 30, changing portfolio weights, and a worst-case scenario for daily





volatility and correlations), and it can be seen that by day 10 the methodology still correctly identifies on average 125.8 = 130 - 4.2 of the stocks in the portfolio, and by day 10 the methodology only incorrectly includes in the portfolio on average 16.2 of the 870 stocks that are not in the portfolio. Moreover, it can be seen from Table 2.3 that if the best-case scenario for daily volatility and correlations is considered with everything else remaining unchanged (130 stocks, scaled price of 30, and changing portfolio weights), then by day 9 the methodology has always correctly identified all 130 stocks in the portfolio, and by day 6 the methodology has always correctly rejected all 870 stocks that are not in the portfolio.

In contrast, for the reporting of prices at time points 1-second apart, the easiest scenario is presented in Table 3.3 (80 stocks, scaled price of 50, constant portfolio weights, and a best-case scenario for daily volatility and correlations), and it can be seen that the methodology has always correctly identified all 80 stocks in the portfolio on the first day, and the methodology has correctly rejected all 920 stocks that are not in the portfolio on either the first or the second day.

For the reporting of prices at time points 15-seconds apart, the most difficult scenario is presented in Table 6.1 (130 stocks, scaled price of 30, changing portfolio weights, and a worst-case scenario for daily volatility and correlations), and it can be seen that by day 30 the methodology still correctly identifies on average 126.4 = 130 - 3.6 of the stocks in the portfolio, and by day 30 the methodology only incorrectly includes in the portfolio on average



BLUE TRACTOR



18.6 of the 870 stocks that are not in the portfolio. Moreover, it can be seen from Table 6.3 that if the best-case scenario for daily volatility and correlations is considered with everything else remaining unchanged (130 stocks, scaled price of 30, and changing portfolio weights), then by day 21 the methodology has always correctly identified all 130 stocks in the portfolio, and by day 15 the methodology has always correctly rejected all 870 stocks that are not in the portfolio.

Again, in contrast, for the reporting of prices at time points 15-seconds apart, the easiest scenario is presented in Table 7.3 (80 stocks, scaled price of 50, constant portfolio weights, and a best-case scenario for daily volatility and correlations), and it can be seen that the methodology has by day 3 always correctly identified all 80 stocks in the portfolio, and the methodology has correctly rejected all 920 stocks that are not in the portfolio either by day 3 or by day 6.

It should also be noted that while the simulation results presented here are for ten time points (10 days for the reporting of prices at time points 1-second apart and 30 days for the reporting of prices at time points 15-seconds apart), the methodology may be run for longer time periods. It would be expected that, in general, more accurate results would be obtainable for longer time periods.

Finally, it is important to remember that, in practice, it would be expected that experts with knowledge of the specific stocks involved, with some prior historical information about the portfolio, and with an





understanding of prevailing market conditions, for example, would be able to fine tune a methodology such as the multi-day methodology presented here in order to substantially improve its performance.

Consequently, the results presented in my initial report and in this supplemental report are intended to demonstrate the practicality of reverse engineering for the scenarios considered, but they are not intended to demonstrate any optimal approach to the problem of reverse engineering nor to provide any optimal results. As explained, it would be expected that superior results to the simulation results that I have presented could be obtained in practice by any serious attempt at reverse engineering.



	Table 1.1							
Universe of k =	Universe of k = 1,000 potential stocks. Portfolio initially contains 130 stocks with equal weights 1/130.							
Initial scale	d value of a portfolio sha	re = 50. Portfolio share ¡	price rounded to r=2 dec	imal places.				
	1-second	reporting (23,400 values	per day).					
Wor	st-case scenario for daily	volatility and correlation	ns: $\sigma_d$ = 0.0137 and $ ho$ = 0	.551.				
	Table entries are average values based on M = 10 simulations.							
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.				
	Po	rtfolio weights unchange	ed.					
Day 1	33.9	0.0035	14.2	0.1007				
Day 2	6.2	0.0017	11.2	0.0546				
Day 3	7.5	0.0016	0.7	0.0039				
Day 4	0.9	0.0011	0.5	0.0025				
Day 5	0.1	0.0010	0.5	0.0021				
Day 6	0.0	0.0009	0.0	0.0000				
Day 7	0.0	0.0008	0.1	0.0004				
Day 8	0.0	0.0008	0.3	0.0011				
Day 9	0.0	0.0007	0.0	0.0000				
Day 10	0.0	0.0007	0.0	0.0000				
	Portfo	olio weights change day t	o day.					
Day 1	34.6	0.0035	15.9	0.1133				
Day 2	6.8	0.0017	10.9	0.0544				
Day 3	7.1	0.0016	0.8	0.0046				
Day 4	0.9	0.0011	0.5	0.0023				
Day 5	0.2	0.0010	0.7	0.0029				
Day 6	0.3	0.0009	0.0	0.0000				
Day 7	0.0	0.0008	0.3	0.0012				
Day 8	0.0	0.0008	0.1	0.0004				
Day 9	0.0	0.0008	0.0	0.0000				
Day 10	0.0	0.0007	0.0	0.0000				



	Table 1.2							
Universe of k	= 1,000 potential stocks.	Portfolio initially contain	ns 130 stocks with equal	weights 1/130.				
Initial scale	d value of a portfolio sha	are = 50. Portfolio share	price rounded to r=2 dec	imal places.				
	1-second	reporting (23,400 values	per day).					
Ave	rage scenario for daily vo	olatility and correlations:	$\sigma_d$ = 0.0173 and $ ho$ = 0.	278.				
	Table entries are average values based on M = 10 simulations.							
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.				
	Po	ortfolio weights unchange	ed.					
Day 1	6.8	0.0016	2.6	0.0138				
Day 2	0.1	0.0010	0.5	0.0022				
Day 3	0.0	0.0008	0.0	0.0000				
Day 4	0.0	0.0007	0.0	0.0000				
Day 5	0.0	0.0006	0.0	0.0000				
Day 6	0.0	0.0006	0.0	0.0000				
Day 7	0.0	0.0005	0.0	0.0000				
Day 8	0.0	0.0005	0.0	0.0000				
Day 9	0.0	0.0005	0.0	0.0000				
Day 10	0.0	0.0004	0.0	0.0000				
	Portfo	olio weights change day t	o day.					
Day 1	6.2	0.0016	3.3	0.0177				
Day 2	0.2	0.0010	0.9	0.0037				
Day 3	0.2	0.0008	0.0	0.0000				
Day 4	0.0	0.0007	0.0	0.0000				
Day 5	0.0	0.0007	0.0	0.0000				
Day 6	0.0	0.0006	0.0	0.0000				
Day 7	0.0	0.0006	0.0	0.0000				
Day 8	0.0	0.0006	0.0	0.0000				
Day 9	0.0	0.0005	0.0	0.0000				
Day 10	0.0	0.0005	0.0	0.0000				



		Table 1.3						
Universe of k =	= 1,000 potential stocks.	Portfolio initially contain	ns 130 stocks with equal	weights 1/130.				
Initial scale	d value of a portfolio sha	re = 50. Portfolio share	price rounded to r=2 dec	imal places.				
	1-second	reporting (23,400 values	per day).					
Best	-case scenario for daily v	olatility and correlations	: $\sigma_d$ = 0.0237 and $\rho$ = 0	.181.				
	Table entries are average values based on M = 10 simulations.							
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.				
	Po	ortfolio weights unchange	ed.					
Day 1	0.3	0.0010	0.3	0.0013				
Day 2	0.0	0.0007	0.0	0.0000				
Day 3	0.0	0.0005	0.0	0.0000				
Day 4	0.0	0.0005	0.0	0.0000				
Day 5	0.0	0.0004	0.0	0.0000				
Day 6	0.0	0.0004	0.0	0.0000				
Day 7	0.0	0.0004	0.0	0.0000				
Day 8	0.0	0.0003	0.0	0.0000				
Day 9	0.0	0.0003	0.0	0.0000				
Day 10	0.0	0.0003	0.0	0.0000				
	Portfo	olio weights change day t	o day.					
Day 1	0.3	0.0009	0.1	0.0005				
Day 2	0.0	0.0006	0.1	0.0004				
Day 3	0.0	0.0005	0.0	0.0000				
Day 4	0.0	0.0005	0.0	0.0000				
Day 5	0.0	0.0005	0.0	0.0000				
Day 6	0.0	0.0004	0.0	0.0000				
Day 7	0.0	0.0004	0.0	0.0000				
Day 8	0.0	0.0004	0.0	0.0000				
Day 9	0.0	0.0004	0.0	0.0000				
Day 10	0.0	0.0004	0.0	0.0000				



		Table 2.1						
Universe of k	Universe of k = 1,000 potential stocks. Portfolio initially contains 130 stocks with equal weights 1/130.							
Initial scale	d value of a portfolio sha	re = 30. Portfolio share	price rounded to r=2 dec	imal places.				
	1-second	reporting (23,400 values	per day).					
Wor	st-case scenario for daily	volatility and correlation	ns: $\sigma_d$ = 0.0137 and $ ho$ = 0.	.551.				
	Table entries are average values based on M = 10 simulations.							
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.				
	Po	rtfolio weights unchange	ed.					
Day 1	77.6	0.0058	48.0	0.4459				
Day 2	45.9	0.0038	63.7	0.3651				
Day 3	57.4	0.0060	10.7	0.1094				
Day 4	30.3	0.0030	23.5	0.1510				
Day 5	15.9	0.0022	36.7	0.1782				
Day 6	21.6	0.0025	8.9	0.0596				
Day 7	9.1	0.0017	17.6	0.0927				
Day 8	5.1	0.0015	28.0	0.1245				
Day 9	7.4	0.0015	9.2	0.0502				
Day 10	3.7	0.0013	16.9	0.0794				
	Portfo	olio weights change day t	o day.					
Day 1	76.9	0.0056	51.7	0.4587				
Day 2	44.2	0.0038	62.6	0.3644				
Day 3	61.0	0.0061	13.8	0.1427				
Day 4	33.2	0.0032	21.6	0.1468				
Day 5	18.5	0.0024	39.0	0.1931				
Day 6	28.0	0.0031	7.8	0.0559				
Day 7	13.1	0.0020	18.9	0.1021				
Day 8	6.5	0.0017	29.8	0.1338				
Day 9	9.8	0.0017	8.6	0.0485				
Day 10	4.2	0.0015	16.2	0.0769				







		Table 2.2						
Universe of k =	Universe of k = 1,000 potential stocks. Portfolio initially contains 130 stocks with equal weights 1/130.							
Initial scale	d value of a portfolio sha	re = 30. Portfolio share ¡	price rounded to r=2 dec	imal places.				
	1-second	reporting (23,400 values	per day).					
Ave				278.				
	Average scenario for daily volatility and correlations: $\sigma_d$ = 0.0173 and $\rho$ = 0.278.  Table entries are average values based on M = 10 simulations.							
	Number of stocks incorrectly excluded from the estimated portfolio.  Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.  Number of stocks incorrectly included in the estimated portfolio.  Sum of estimated weights for stocks incorrectly included in the estimated portfolio.  the estimated portfolio.							
	Po	rtfolio weights unchange	ed.					
Day 1	55.0	0.0047	28.3	0.2369				
Day 2	21.0	0.0026	29.0	0.1591				
Day 3	28.0	0.0032	4.2	0.0307				
Day 4	8.6	0.0017	5.4	0.0311				
Day 5	1.8	0.0013	9.7	0.0450				
Day 6	4.5	0.0014	1.2	0.0066				
Day 7	0.7	0.0011	2.6	0.0123				
Day 8	0.0	0.0010	3.1	0.0131				
Day 9	0.5	0.0010	0.5	0.0024				
Day 10	0.1	0.0009	1.3	0.0056				
	Portfo	lio weights change day t	o day.					
Day 1	55.6	0.0048	27.9	0.2370				
Day 2	19.9	0.0025	27.4	0.1492				
Day 3	28.6	0.0033	2.8	0.0213				
Day 4	10.8	0.0019	4.9	0.0280				
Day 5	2.3	0.0014	8.5	0.0393				
Day 6	4.6	0.0014	1.0	0.0054				
Day 7	0.9	0.0012	2.1	0.0101				
Day 8	0.3	0.0011	3.3	0.0137				
Day 9	0.8	0.0010	0.2	0.0009				
Day 10	0.3	0.0010	0.8	0.0033				







		Table 2.3						
Universe of k	Universe of k = 1,000 potential stocks. Portfolio initially contains 130 stocks with equal weights 1/130.							
Initial scale	d value of a portfolio sha	re = 30. Portfolio share	price rounded to r=2 dec	imal places.				
	1-second	reporting (23,400 values	per day).					
Best	-case scenario for daily v	olatility and correlations	: $\sigma_d$ = 0.0237 and $\rho$ = 0.	.181.				
	Table entries are average values based on M = 10 simulations.							
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.				
	Po	ortfolio weights unchange	ed.					
Day 1	24.6	0.0029	9.5	0.0627				
Day 2	3.0	0.0015	5.5	0.0257				
Day 3	4.3	0.0013	0.1	0.0006				
Day 4	0.5	0.0010	0.2	0.0009				
Day 5	0.0	0.0009	0.1	0.0004				
Day 6	0.0	0.0008	0.0	0.0000				
Day 7	0.0	0.0007	0.0	0.0000				
Day 8	0.0	0.0007	0.0	0.0000				
Day 9	0.0	0.0007	0.0	0.0000				
Day 10	0.0	0.0006	0.0	0.0000				
	Portfo	olio weights change day t	o day.					
Day 1	23.9	0.0028	10.7	0.0706				
Day 2	3.3	0.0014	6.2	0.0294				
Day 3	3.7	0.0013	0.0	0.0000				
Day 4	0.5	0.0010	0.2	0.0009				
Day 5	0.1	0.0009	0.3	0.0012				
Day 6	0.1	0.0008	0.0	0.0000				
Day 7	0.1	0.0008	0.0	0.0000				
Day 8	0.1	0.0007	0.0	0.0000				
Day 9	0.0	0.0007	0.0	0.0000				
Day 10	0.0	0.0007	0.0	0.0000				







	Table 3.1							
Universe of k	Universe of k = 1,000 potential stocks. Portfolio initially contains 80 stocks with equal weights 1/80.							
Initial scale	d value of a portfolio sha	re = 50. Portfolio share ¡	price rounded to r=2 dec	imal places.				
	1-second	reporting (23,400 values	per day).					
Wor	st-case scenario for daily	volatility and correlation	ns: $\sigma_d$ = 0.0137 and $ ho$ = 0	.551.				
	Table entries are average values based on M = 10 simulations.							
	Number of stocks incorrectly excluded from the estimated portfolio.  Average absolute difference between true weight and estimated weight for the 80 stocks in the portfolio.  Average absolute difference between true weight and estimated weight for the 80 stocks in the portfolio.  Number of stocks incorrectly included in the estimated portfolio.  Sum of estimated weights for stocks incorrectly included in the estimated portfolio.							
	Po	rtfolio weights unchange	ed.					
Day 1	0.7	0.0022	11.6	0.0742				
Day 2	0.0	0.0015	7.5	0.0380				
Day 3	0.0	0.0013	0.3	0.0016				
Day 4	0.0	0.0011	0.4	0.0019				
Day 5	0.0	0.0010	0.6	0.0026				
Day 6	0.0	0.0009	0.1	0.0005				
Day 7	0.0	0.0008	0.0	0.0000				
Day 8	0.0	0.0007	0.2	0.0008				
Day 9	0.0	0.0007	0.0	0.0000				
Day 10	0.0	0.0007	0.0	0.0000				
	Portfo	olio weights change day t	o day.					
Day 1	0.8	0.0022	12.0	0.0778				
Day 2	0.0	0.0016	8.2	0.0397				
Day 3	0.0	0.0013	0.3	0.0016				
Day 4	0.0	0.0011	0.3	0.0014				
Day 5	0.0	0.0010	0.4	0.0017				
Day 6	0.0	0.0010	0.0	0.0000				
Day 7	0.0	0.0010	0.0	0.0000				
Day 8	0.0	0.0009	0.0	0.0000				
Day 9	0.0	0.0009	0.0	0.0000				
Day 10	0.0	0.0009	0.0	0.0000				







		Table 3.2						
Universe of k = 1,000 potential stocks. Portfolio initially contains 80 stocks with equal weights 1/80.  Initial scaled value of a portfolio share = 50. Portfolio share price rounded to r=2 decimal places.								
							1-second reporting (23,400 values per day).	
Avo	2 2 2							
Average scenario for daily volatility and correlations: $\sigma_d$ = 0.0173 and $\rho$ = 0.278.  Table entries are average values based on M = 10 simulations.								
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 80 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.				
	Portfolio weights unchanged.							
Day 1	0	0.0014	2.0	0.0106				
Day 2	0	0.0010	0.5	0.0022				
Day 3	0	0.0008	0.0	0.0000				
Day 4	0	0.0007	0.0	0.0000				
Day 5	0	0.0006	0.0	0.0000				
Day 6	0	0.0006	0.0	0.0000				
Day 7	0	0.0005	0.0	0.0000				
Day 8	0	0.0005	0.0	0.0000				
Day 9	0	0.0005	0.0	0.0000				
Day 10	0	0.0005	0.0	0.0000				
	Portfo	olio weights change day t	o day.					
Day 1	0	0.0014	3.0	0.0161				
Day 2	0	0.0010	1.1	0.0046				
Day 3	0	0.0009	0.0	0.0000				
Day 4	0	0.0008	0.0	0.0000				
Day 5	0	0.0007	0.0	0.0000				
Day 6	0	0.0007	0.0	0.0000				
Day 7	0	0.0007	0.0	0.0000				
Day 8	0	0.0007	0.0	0.0000				
Day 9	0	0.0007	0.0	0.0000				
Day 10	0	0.0007	0.0	0.0000				



	Table 3.3														
Universe of k = 1,000 potential stocks. Portfolio initially contains 80 stocks with equal weights 1/80. Initial scaled value of a portfolio share = 50. Portfolio share price rounded to r=2 decimal places.   1-second reporting (23,400 values per day).  Best-case scenario for daily volatility and correlations: $\sigma_d$ = 0.0237 and $\rho$ = 0.181.															
								Table entries are average values based on M = 10 simulations.							
									Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 80 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.			
									Portfolio weights unchanged.						
Day 1	0	0.0009	0.2	0.0009											
Day 2	0	0.0007	0.0	0.0000											
Day 3	0	0.0005	0.0	0.0000											
Day 4	0	0.0005	0.0	0.0000											
Day 5	0	0.0004	0.0	0.0000											
Day 6	0	0.0004	0.0	0.0000											
Day 7	0	0.0003	0.0	0.0000											
Day 8	0	0.0003	0.0	0.0000											
Day 9	0	0.0003	0.0	0.0000											
Day 10	0	0.0003	0.0	0.0000											
	Portfo	olio weights change day t	o day.												
Day 1	0	0.0009	0.1	0.0005											
Day 2	0	0.0007	0.0	0.0000											
Day 3	0	0.0005	0.0	0.0000											
Day 4	0	0.0005	0.0	0.0000											
Day 5	0	0.0005	0.0	0.0000											
Day 6	0	0.0005	0.0	0.0000											
Day 7	0	0.0006	0.0	0.0000											
Day 8	0	0.0006	0.0	0.0000											
Day 9	0	0.0006	0.0	0.0000											
Day 10	0	0.0006	0.0	0.0000											







	Table 4.1  Universe of k = 1,000 potential stocks. Portfolio initially contains 80 stocks with equal weights 1/80.							
Universe of k								
Initial scaled value of a portfolio share = 30. Portfolio share price rounded to r=2 decimal places.								
	1-second reporting (23,400 values per day).							
Worst-case scenario for daily volatility and correlations: $\sigma_d$ = 0.0137 and $ ho$ = 0.551.								
Table entries are average values based on M = 10 simulations.								
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 80 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.				
Portfolio weights unchanged.								
Day 1	25.5	0.0060	50.6	0.4191				
Day 2	6.0	0.0044	59.6	0.3301				
Day 3	6.6	0.0031	10.8	0.0901				
Day 4	0.6	0.0024	20.8	0.1294				
Day 5	0.1	0.0024	33.6	0.1666				
Day 6	0.2	0.0018	7.8	0.0505				
Day 7	0.0	0.0018	17.4	0.0921				
Day 8	0.0	0.0019	28.7	0.1281				
Day 9	0.0	0.0015	8.3	0.0453				
Day 10	0.0	0.0015	16.8	0.0799				
	Portfo	olio weights change day t	o day.					
Day 1	23.8	0.0057	44.6	0.3846				
Day 2	5.7	0.0042	53.7	0.3067				
Day 3	6.9	0.0031	9.8	0.0830				
Day 4	1.5	0.0025	22.3	0.1371				
Day 5	0.5	0.0025	34.4	0.1700				
Day 6	0.4	0.0019	7.3	0.0468				
Day 7	0.0	0.0019	17.0	0.0899				
Day 8	0.0	0.0020	27.3	0.1224				
Day 9	0.0	0.0016	8.2	0.0451				
Day 10	0.0	0.0016	15.1	0.0724				







		Table 4.2		
Universe of k	= 1,000 potential stocks	. Portfolio initially conta	ins 80 stocks with equal	weights 1/80.
Initial scale	d value of a portfolio sha	re = 30. Portfolio share	price rounded to r=2 dec	imal places.
	1-second	reporting (23,400 values	per day).	
Ave	rage scenario for daily vo	platility and correlations:	$\sigma_d$ = 0.0173 and $ ho$ = 0.	278.
	Table entries are a	verage values based on I	M = 10 simulations.	
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 80 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.
	Po	ortfolio weights unchange	ed.	
Day 1	6.2	0.0034	29.3	0.2042
Day 2	0.3	0.0024	27.2	0.1420
Day 3	0.2	0.0018	2.8	0.0183
Day 4	0.0	0.0016	5.2	0.0277
Day 5	0.0	0.0014	8.5	0.0386
Day 6	0.0	0.0012	1.0	0.0053
Day 7	0.0	0.0011	2.4	0.0110
Day 8	0.0	0.0011	3.6	0.0152
Day 9	0.0	0.0010	0.2	0.0010
Day 10	0.0	0.0010	0.8	0.0034
	Portfo	olio weights change day t	o day.	
Day 1	7.3	0.0036	24.9	0.1856
Day 2	0.3	0.0023	23.3	0.1232
Day 3	0.0	0.0017	2.3	0.0150
Day 4	0.0	0.0016	4.3	0.0229
Day 5	0.0	0.0014	7.9	0.0366
Day 6	0.0	0.0013	0.8	0.0045
Day 7	0.0	0.0012	2.3	0.0113
Day 8	0.0	0.0012	4.4	0.0190
Day 9	0.0	0.0012	0.8	0.0039
Day 10	0.0	0.0011	1.0	0.0044





		Table 4.3		
Universe of k	= 1,000 potential stocks.	. Portfolio initially conta	ins 80 stocks with equal	weights 1/80.
Initial scale	d value of a portfolio sha	re = 30. Portfolio share	price rounded to r=2 dec	imal places.
	1-second	reporting (23,400 values	per day).	
Best	-case scenario for daily v	olatility and correlations	: $\sigma_d$ = 0.0237 and $ ho$ = 0.	.181.
	Table entries are a	verage values based on I	M = 10 simulations.	
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 80 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.
	Po	ortfolio weights unchange	ed.	
Day 1	0.3	0.0020	8.9	0.0546
Day 2	0.0	0.0014	5.7	0.0266
Day 3	0.0	0.0011	0.3	0.0015
Day 4	0.0	0.0010	0.2	0.0009
Day 5	0.0	0.0009	0.3	0.0012
Day 6	0.0	0.0008	0.0	0.0000
Day 7	0.0	0.0008	0.0	0.0000
Day 8	0.0	0.0007	0.0	0.0000
Day 9	0.0	0.0006	0.0	0.0000
Day 10	0.0	0.0006	0.0	0.0000
	Portfo	olio weights change day t	o day.	
Day 1	0.1	0.0019	8.4	0.0513
Day 2	0.0	0.0013	5.0	0.0240
Day 3	0.0	0.0011	0.1	0.0005
Day 4	0.0	0.0010	0.3	0.0014
Day 5	0.0	0.0009	0.1	0.0004
Day 6	0.0	0.0009	0.0	0.0000
Day 7	0.0	0.0009	0.0	0.0000
Day 8	0.0	0.0008	0.0	0.0000
Day 9	0.0	0.0008	0.0	0.0000
Day 10	0.0	0.0008	0.0	0.0000



		Table 5.1		
Universe of k	= 1,000 potential stocks.	Portfolio initially contain	ns 130 stocks with equal	weights 1/130.
Initial scale	d value of a portfolio sha	re = 50. Portfolio share	price rounded to r=2 dec	imal places.
	15-secon	d reporting (1,560 values	per day).	
Wor	st-case scenario for daily	volatility and correlation	ns: $\sigma_d$ = 0.0137 and $ ho$ = 0.	.551.
	Table entries are a	verage values based on I	M = 10 simulations.	
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.
	Po	ortfolio weights unchange	ed.	
Day 3	48.2	0.0044	23.4	0.1878
Day 6	16.1	0.0023	20.8	0.1107
Day 9	19.9	0.0026	2.1	0.0144
Day 12	5.4	0.0015	2.8	0.0151
Day 15	1.3	0.0012	4.1	0.0185
Day 18	2.3	0.0012	0.5	0.0026
Day 21	0.0	0.0010	1.0	0.0046
Day 24	0.0	0.0009	1.7	0.0070
Day 27	0.2	0.0009	0.1	0.0004
Day 30	0.0	0.0008	0.3	0.0012
	Portfol	io weights change every	3 days.	
Day 3	49.4	0.0045	22.7	0.1906
Day 6	16.0	0.0023	20.3	0.1098
Day 9	18.7	0.0025	2.1	0.0139
Day 12	4.6	0.0015	3.2	0.0170
Day 15	0.9	0.0012	4.9	0.0219
Day 18	2.5	0.0012	0.2	0.0010
Day 21	0.5	0.0010	1.1	0.0049
Day 24	0.1	0.0009	2.3	0.0091
Day 27	0.1	0.0009	0.2	0.0009
Day 30	0.0	0.0009	0.3	0.0012



		Table 5.2		
Universe of k	= 1,000 potential stocks.	Portfolio initially contair	ns 130 stocks with equal	weights 1/130.
Initial scale	d value of a portfolio sha	re = 50. Portfolio share	price rounded to r=2 dec	imal places.
	15-second	d reporting (1,560 values	per day).	
Ave	rage scenario for daily vo	platility and correlations:	$\sigma_d$ = 0.0173 and $ ho$ = 0.	278.
	Table entries are a	verage values based on I	M = 10 simulations.	
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.
	Po	ortfolio weights unchange	ed.	
Day 3	18.0	0.0025	7.2	0.0439
Day 6	1.4	0.0013	3.2	0.0147
Day 9	1.1	0.0010	0.1	0.0005
Day 12	0.0	0.0009	0.0	0.0000
Day 15	0.0	0.0008	0.1	0.0004
Day 18	0.0	0.0007	0.0	0.0000
Day 21	0.0	0.0006	0.0	0.0000
Day 24	0.0	0.0006	0.0	0.0000
Day 27	0.0	0.0006	0.0	0.0000
Day 30	0.0	0.0005	0.0	0.0000
	Portfol	io weights change every	3 days.	
Day 3	14.8	0.0022	6.1	0.0376
Day 6	1.1	0.0012	2.6	0.0117
Day 9	1.0	0.0010	0.0	0.0000
Day 12	0.0	0.0009	0.0	0.0000
Day 15	0.0	0.0008	0.0	0.0000
Day 18	0.0	0.0007	0.0	0.0000
Day 21	0.0	0.0007	0.0	0.0000
Day 24	0.0	0.0007	0.0	0.0000
Day 27	0.0	0.0006	0.0	0.0000
Day 30	0.0	0.0006	0.0	0.0000







Table 5.3					
Universe of k =	Universe of k = 1,000 potential stocks. Portfolio initially contains 130 stocks with equal weights 1/130.				
Initial scale	d value of a portfolio sha	re = 50. Portfolio share	price rounded to r=2 dec	imal places.	
	15-secon	d reporting (1,560 values	per day).		
Best	-case scenario for daily v	olatility and correlations	: $\sigma_d$ = 0.0237 and $\rho$ = 0.	.181.	
	Table entries are a	verage values based on I	VI = 10 simulations.		
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.	
	Po	ortfolio weights unchange	ed.		
Day 3	2.1	0.0013	1.1	0.0054	
Day 6	0.0	0.0008	0.2	0.0008	
Day 9	0.0	0.0007	0.0	0.0000	
Day 12	0.0	0.0006	0.0	0.0000	
Day 15	0.0	0.0005	0.0	0.0000	
Day 18	0.0	0.0005	0.0	0.0000	
Day 21	0.0	0.0004	0.0	0.0000	
Day 24	0.0	0.0004	0.0	0.0000	
Day 27	0.0	0.0004	0.0	0.0000	
Day 30	0.0	0.0004	0.0	0.0000	
	Portfol	io weights change every	3 days.		
Day 3	2.6	0.0013	0.9	0.0044	
Day 6	0.0	0.0008	0.0	0.0000	
Day 9	0.0	0.0007	0.0	0.0000	
Day 12	0.0	0.0006	0.0	0.0000	
Day 15	0.0	0.0006	0.0	0.0000	
Day 18	0.0	0.0005	0.0	0.0000	
Day 21	0.0	0.0005	0.0	0.0000	
Day 24	0.0	0.0005	0.0	0.0000	
Day 27	0.0	0.0005	0.0	0.0000	
Day 30	0.0	0.0005	0.0	0.0000	



	Table 6.1				
Universe of k =	Universe of k = 1,000 potential stocks. Portfolio initially contains 130 stocks with equal weights 1/130.				
Initial scale	d value of a portfolio sha	re = 30. Portfolio share	price rounded to r=2 dec	imal places.	
	15-second	d reporting (1,560 values	per day).		
Wor	st-case scenario for daily	volatility and correlation	ns: $\sigma_d$ = 0.0137 and $ ho$ = 0	.551.	
	Table entries are a	verage values based on I	VI = 10 simulations.		
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.	
	Po	ortfolio weights unchange	ed.		
Day 3	78.3	0.0059	48.1	0.4470	
Day 6	46.8	0.0039	61.5	0.3629	
Day 9	58.9	0.0061	11.2	0.1193	
Day 12	31.4	0.0031	22.6	0.1502	
Day 15	16.7	0.0023	37.2	0.1833	
Day 18	26.0	0.0029	9.2	0.0644	
Day 21	11.6	0.0019	19.8	0.1053	
Day 24	6.3	0.0017	28.1	0.1254	
Day 27	9.6	0.0017	7.1	0.0401	
Day 30	3.5	0.0014	16.6	0.0771	
	Portfol	io weights change every	3 days.		
Day 3	74.3	0.0055	47.2	0.4290	
Day 6	43.9	0.0038	60.7	0.3532	
Day 9	59.3	0.0060	12.9	0.1359	
Day 12	32.9	0.0032	22.0	0.1496	
Day 15	15.4	0.0023	33.2	0.1678	
Day 18	25.1	0.0029	8.3	0.0562	
Day 21	10.1	0.0019	19.1	0.0996	
Day 24	5.7	0.0017	29.7	0.1305	
Day 27	7.8	0.0017	8.7	0.0480	
Day 30	3.6	0.0015	18.6	0.0864	



Table 6.2					
Universe of k =	Universe of k = 1,000 potential stocks. Portfolio initially contains 130 stocks with equal weights 1/130.				
Initial scale	d value of a portfolio sha	re = 30. Portfolio share	price rounded to r=2 dec	imal places.	
	15-second	d reporting (1,560 values	per day).		
Ave	rage scenario for daily vo	platility and correlations:	$\sigma_d$ = 0.0173 and $\rho$ = 0.	278.	
	Table entries are a	verage values based on I	VI = 10 simulations.		
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.	
	Po	rtfolio weights unchange	ed.		
Day 3	52.6	0.0047	25.9	0.2147	
Day 6	19.1	0.0025	23.3	0.1290	
Day 9	24.6	0.0030	2.0	0.0150	
Day 12	7.6	0.0017	3.6	0.0205	
Day 15	2.2	0.0014	6.3	0.0288	
Day 18	3.3	0.0013	0.6	0.0032	
Day 21	1.0	0.0011	1.4	0.0065	
Day 24	0.1	0.0010	2.2	0.0093	
Day 27	0.2	0.0010	0.4	0.0018	
Day 30	0.0	0.0009	0.5	0.0021	
	Portfol	io weights change every	3 days.		
Day 3	49.1	0.0044	24.1	0.1916	
Day 6	16.3	0.0023	23.6	0.1276	
Day 9	21.8	0.0027	2.4	0.0166	
Day 12	7.3	0.0016	3.1	0.0164	
Day 15	1.8	0.0013	4.5	0.0200	
Day 18	3.5	0.0013	0.4	0.0021	
Day 21	0.6	0.0011	0.5	0.0023	
Day 24	0.4	0.0010	1.4	0.0059	
Day 27	0.8	0.0010	0.1	0.0005	
Day 30	0.0	0.0009	0.2	0.0009	



		Table 6.3		
Universe of k	= 1,000 potential stocks.	Portfolio initially contair	ns 130 stocks with equal	weights 1/130.
Initial scale	d value of a portfolio sha	re = 30. Portfolio share	price rounded to r=2 dec	imal places.
	15-second	d reporting (1,560 values	per day).	
Best	-case scenario for daily v	olatility and correlations	: $\sigma_d$ = 0.0237 and $ ho$ = 0	.181.
	Table entries are a	verage values based on I	VI = 10 simulations.	
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 130 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.
	Po	rtfolio weights unchange	ed.	
Day 3	23.2	0.0027	12.4	0.0815
Day 6	2.8	0.0014	4.9	0.0230
Day 9	3.0	0.0012	0.2	0.0010
Day 12	0.3	0.0010	0.0	0.0000
Day 15	0.0	0.0009	0.1	0.0004
Day 18	0.1	0.0008	0.0	0.0000
Day 21	0.0	0.0007	0.1	0.0004
Day 24	0.0	0.0007	0.1	0.0004
Day 27	0.0	0.0006	0.0	0.0000
Day 30	0.0	0.0006	0.0	0.0000
	Portfol	io weights change every	3 days.	
Day 3	25.2	0.0029	10.8	0.0728
Day 6	2.6	0.0014	6.6	0.0314
Day 9	2.5	0.0012	0.1	0.0005
Day 12	0.2	0.0010	0.1	0.0005
Day 15	0.0	0.0009	0.0	0.0000
Day 18	0.1	0.0008	0.0	0.0000
Day 21	0.0	0.0007	0.0	0.0000
Day 24	0.0	0.0007	0.0	0.0000
Day 27	0.0	0.0007	0.0	0.0000
Day 30	0.0	0.0007	0.0	0.0000







		Table 7.1		
Universe of k	x = 1,000 potential stocks	. Portfolio initially conta	ins 80 stocks with equal	weights 1/80.
Initial scale	d value of a portfolio sha	are = 50. Portfolio share	price rounded to r=2 dec	imal places.
	15-secon	d reporting (1,560 values	per day).	
Wor	rst-case scenario for daily	volatility and correlation	ns: $\sigma_d$ = 0.0137 and $\rho$ = 0.	551.
	Table entries are a	verage values based on I	M = 10 simulations.	
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 80 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.
	Po	ortfolio weights unchange	ed.	
Day 3	3.7	0.0030	23.2	0.1578
Day 6	0.0	0.0021	20.7	0.1066
Day 9	0.0	0.0015	1.6	0.0099
Day 12	0.0	0.0013	2.1	0.0111
Day 15	0.0	0.0012	3.4	0.0151
Day 18	0.0	0.0011	0.6	0.0030
Day 21	0.0	0.0010	1.0	0.0045
Day 24	0.0	0.0009	1.4	0.0057
Day 27	0.0	0.0009	0.1	0.0004
Day 30	0.0	0.0008	0.4	0.0017
	Portfol	lio weights change every	3 days.	
Day 3	4.4	0.0030	21.8	0.1527
Day 6	0.0	0.0021	20.2	0.1051
Day 9	0.0	0.0016	1.3	0.0080
Day 12	0.0	0.0014	2.8	0.0146
Day 15	0.0	0.0013	3.8	0.0176
Day 18	0.0	0.0012	0.1	0.0006
Day 21	0.0	0.0011	0.6	0.0027
Day 24	0.0	0.0011	1.2	0.0048
Day 27	0.0	0.0010	0.1	0.0004
Day 30	0.0	0.0010	0.5	0.0020



		Table 7.2		
Universe of k	= 1,000 potential stocks		ins 80 stocks with equal	weights 1/80.
Initial scale	d value of a portfolio sha	re = 50. Portfolio share p	price rounded to r=2 dec	imal places.
	15-second	d reporting (1,560 values	per day).	
Ave	rage scenario for daily vo	platility and correlations:	$\sigma_d = 0.0173$ and $\rho = 0.3$	278.
	Table entries are a	verage values based on N	M = 10 simulations.	
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 80 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.
	Po	rtfolio weights unchange	ed.	
Day 3	0.1	0.0017	5.9	0.0342
Day 6	0.0	0.0012	2.9	0.0132
Day 9	0.0	0.0010	0.2	0.0010
Day 12	0.0	0.0009	0.2	0.0009
Day 15	0.0	0.0008	0.3	0.0012
Day 18	0.0	0.0007	0.0	0.0000
Day 21	0.0	0.0006	0.0	0.0000
Day 24	0.0	0.0006	0.0	0.0000
Day 27	0.0	0.0006	0.0	0.0000
Day 30	0.0	0.0005	0.0	0.0000
	Portfol	io weights change every	3 days.	
Day 3	0	0.0017	5.6	0.0334
Day 6	0	0.0012	1.6	0.0072
Day 9	0	0.0010	0.1	0.0005
Day 12	0	0.0009	0.0	0.0000
Day 15	0	0.0009	0.1	0.0004
Day 18	0	0.0008	0.0	0.0000
Day 21	0	0.0008	0.0	0.0000
Day 24	0	0.0008	0.0	0.0000
Day 27	0	0.0008	0.0	0.0000
Day 30	0	0.0008	0.0	0.0000







		Table 7.3		
	x = 1,000 potential stocks	<u> </u>	<u> </u>	
Initial scale	ed value of a portfolio sha	re = 50. Portfolio share	price rounded to r=2 dec	imal places.
	15-secon	d reporting (1,560 values	per day).	
Best	t-case scenario for daily v	olatility and correlations	: $\sigma_d$ = 0.0237 and $ ho$ = 0	.181.
	Table entries are a	verage values based on I	VI = 10 simulations.	
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 80 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.
	Po	ortfolio weights unchange	ed.	
Day 3	0	0.0011	0.4	0.0021
Day 6	0	0.0008	0.0	0.0000
Day 9	0	0.0007	0.0	0.0000
Day 12	0	0.0006	0.0	0.0000
Day 15	0	0.0005	0.0	0.0000
Day 18	0	0.0005	0.0	0.0000
Day 21	0	0.0004	0.0	0.0000
Day 24	0	0.0004	0.0	0.0000
Day 27	0	0.0004	0.0	0.0000
Day 30	0	0.0004	0.0	0.0000
	Portfol	io weights change every	3 days.	
Day 3	0	0.0012	0.9	0.0044
Day 6	0	0.0009	0.1	0.0004
Day 9	0	0.0007	0.0	0.0000
Day 12	0	0.0007	0.0	0.0000
Day 15	0	0.0006	0.0	0.0000
Day 18	0	0.0006	0.0	0.0000
Day 21	0	0.0007	0.0	0.0000
Day 24	0	0.0007	0.0	0.0000
Day 27	0	0.0007	0.0	0.0000
Day 30	0	0.0007	0.0	0.0000







		Table 8.1			
Universe of k	Universe of k = 1,000 potential stocks. Portfolio initially contains 80 stocks with equal weights 1/80.				
Initial scale	d value of a portfolio sha	re = 30. Portfolio share ¡	price rounded to r=2 dec	imal places.	
	15-second	d reporting (1,560 values	per day).		
Wor	st-case scenario for daily	volatility and correlation	ns: $\sigma_d$ = 0.0137 and $ ho$ = 0	.551.	
	Table entries are a	verage values based on N	M = 10 simulations.		
	Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 80 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.	
	Po	rtfolio weights unchange	ed.		
Day 3	22.0	0.0055	46.1	0.3860	
Day 6	5.2	0.0042	54.4	0.3103	
Day 9	5.1	0.0029	12.4	0.0986	
Day 12	0.5	0.0025	23.9	0.1433	
Day 15	0.2	0.0025	34.3	0.1709	
Day 18	0.0	0.0018	6.8	0.0438	
Day 21	0.0	0.0019	17.7	0.0921	
Day 24	0.0	0.0019	26.7	0.1216	
Day 27	0.0	0.0015	8.4	0.0463	
Day 30	0.0	0.0016	17.7	0.0836	
	Portfol	io weights change every	3 days.		
Day 3	24.2	0.0058	46.0	0.3912	
Day 6	5.4	0.0043	58.4	0.3276	
Day 9	6.7	0.0032	10.8	0.0914	
Day 12	1.0	0.0026	21.8	0.1345	
Day 15	0.1	0.0026	33.9	0.1668	
Day 18	0.1	0.0019	7.4	0.0478	
Day 21	0.0	0.0019	15.5	0.0830	
Day 24	0.0	0.0020	27.2	0.1225	
Day 27	0.0	0.0017	9.0	0.0483	
Day 30	0.0	0.0017	17.5	0.0823	



		Table 8.2												
Universe of k = 1,000 potential stocks. Portfolio initially contains 80 stocks with equal weights 1/80.														
Initial scaled value of a portfolio share = 30. Portfolio share price rounded to r=2 decimal places. $15\text{-second reporting (1,560 values per day)}.$ Average scenario for daily volatility and correlations: $\sigma_d$ = 0.0173 and $\rho$ = 0.278.   Table entries are average values based on M = 10 simulations.														
									Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 80 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.		
								Portfolio weights unchanged.						
								Day 3	4.6	0.0031	23.2	0.1639		
Day 6	0.1	0.0020	20.3	0.1052										
Day 9	0.1	0.0017	2.2	0.0137										
Day 12	0.0	0.0014	2.6	0.0135										
Day 15	0.0	0.0013	4.2	0.0194										
Day 18	0.0	0.0011	0.2	0.0011										
Day 21	0.0	0.0011	0.8	0.0037										
Day 24	0.0	0.0010	2.4	0.0099										
Day 27	0.0	0.0009	0.1	0.0004										
Day 30	0.0	0.0009	0.5	0.0020										
	Portfol	io weights change every	3 days.											
Day 3	5.0	0.0031	20.7	0.1504										
Day 6	0.1	0.0022	21.8	0.1153										
Day 9	0.0	0.0017	1.7	0.0109										
Day 12	0.0	0.0014	2.9	0.0155										
Day 15	0.0	0.0013	4.6	0.0214										
Day 18	0.0	0.0012	0.6	0.0031										
Day 21	0.0	0.0012	1.2	0.0057										
Day 24	0.0	0.0011	2.4	0.0104										
Day 27	0.0	0.0011	0.7	0.0032										
Day 30	0.0	0.0011	0.8	0.0034										



Table 8.3														
Universe of k = 1,000 potential stocks. Portfolio initially contains 80 stocks with equal weights 1/80.														
Initial scaled value of a portfolio share = 30. Portfolio share price rounded to r=2 decimal places. $15\text{-second reporting (1,560 values per day)}.$ Best-case scenario for daily volatility and correlations: $\sigma_d$ = 0.0237 and $\rho$ = 0.181.   Table entries are average values based on M = 10 simulations.														
									Number of stocks incorrectly excluded from the estimated portfolio.	Average absolute difference between true weight and estimated weight for the 80 stocks in the portfolio.	Number of stocks incorrectly included in the estimated portfolio.	Sum of estimated weights for stocks incorrectly included in the estimated portfolio.		
								Portfolio weights unchanged.						
								Day 3	0.4	0.0019	7.8	0.0496		
Day 6	0.0	0.0014	6.0	0.0282										
Day 9	0.0	0.0011	0.4	0.0022										
Day 12	0.0	0.0009	0.5	0.0023										
Day 15	0.0	0.0008	0.7	0.0029										
Day 18	0.0	0.0008	0.0	0.0000										
Day 21	0.0	0.0007	0.1	0.0004										
Day 24	0.0	0.0007	0.2	0.0007										
Day 27	0.0	0.0006	0.0	0.0000										
Day 30	0.0	0.0006	0.0	0.0000										
	Portfo	io weights change every	3 days.											
Day 3	0.3	0.0019	9.7	0.0603										
Day 6	0.0	0.0013	4.9	0.0240										
Day 9	0.0	0.0011	0.4	0.0021										
Day 12	0.0	0.0011	0.0	0.0000										
Day 15	0.0	0.0010	0.0	0.0000										
Day 18	0.0	0.0009	0.0	0.0000										
Day 21	0.0	0.0008	0.0	0.0000										
Day 24	0.0	0.0008	0.0	0.0000										
Day 27	0.0	0.0008	0.0	0.0000										
Day 30	0.0	0.0008	0.0	0.0000										







## Section II: Data and Information Considered.

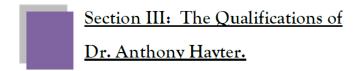
The following data and information have been considered for the preparation of this report.

(1) Precidian's "Fourth Amended and Restated Application" filed with the U.S. Securities and Exchange Commission on September 29, 2017.









I am currently a Full Professor in the Department of Business Information and Analytics at the University of Denver. Between 2006 and 2010 I was the Chair of the Department of Statistics and Operations Technology at the University of Denver, holding the rank of Full Professor.

I have an M.A. in mathematics from Cambridge University, England, scoring a first class in each of my three years there. I obtained my Ph.D. in Statistics from Cornell University at the age of 23. I have spent almost my entire career in an academic environment, and for about thirty years I have held university positions with responsibilities for teaching and researching statistics, probability, and data analysis.

I have established a collaborative research program which has so far resulted in over 90 refereed journal publications, and I have delivered many conference presentations. I have taught a wide range of courses related to statistics, probability, and data analysis at both undergraduate and graduate levels, and I have delivered several keynote addresses at meetings and conferences.





I am the author of the textbook "Probability and Statistics for Engineers and Scientists," the 4th edition of which was published in 2012, and which has been adopted at over sixty universities around the world. I have personally advised eight doctoral students. In addition, I have served as an associate editor of three research journals, and I have presented 88 invited research seminars worldwide.

I have global interests and I have spent considerable time in Japan where I have taught statistics, probability, and data analysis in some Japanese MBA programs. I have received various grants to visit Japanese research institutions and I have also been funded as a visiting researcher in England, Thailand, Singapore, and Hong Kong.

I was awarded a Fulbright Foreign Scholarship Award in 2011-2012 and a Fulbright Specialist Grant in 2014 to assist the government, universities, and businesses in Thailand with surveys, data analysis, curriculum development and research projects.

My full resume is available at HayterStatistics.com.





## Signature Page

I hereby certify that the above report was written by me.

Signed:

Dr. Anthony Hayter

Anthon Master

October 19th, 2017

