Learning from the Market: The Choice Between IPOs and SPAC Mergers*

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December 10, 2023

Abstract

I study when a firm should go public and if it does, whether it should choose IPO or SPAC merger. One of the main benefits of going public is that the firm can make investment decisions based on the information produced by the financial markets. The cost is that the firm has to pay for the information production. By developing a tractable dynamic model, I find that an early-stage firm with more uncertainty about its quality chooses a SPAC because its investment decisions can benefit from the more precise information produced in a SPAC merger. In contrast, a late-stage high-quality firm chooses to go public via IPO because it does not need to pay for the extra information produced in a SPAC merger. Using Simulated Method of Moments, I find that information production costs \$52 (\$121) million in an IPO (SPAC merger). A successful IPO (SPAC merger) increases the firm's value by 4.1% (34.3%), which rises (drops) to 6.7% (17.3%) in a counterfactual setting where SPAC merger (IPO) is chosen instead.

Keywords: Security Issuance, Going Public Decision, Bayesian Learning

*I thank my advisors, Ron Kaniel and Christian Opp, for their generous support. I also thank Gaurab Aryal, Zhaohui Chen, Michael Gofman, Yukun Liu, Toni Whited, Chris Yung, Pavel Zryumov and participants in the PhD workshop in Rochester for their insights and advice. All errors are my own.

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1 Introduction

A large literature explores why and when private firms choose to go public[1]. However, little is known about the decision-making process behind the specific mechanisms firms choose. Since 2016, every one out of three private firms chose Special Purpose Acquisition Company (SPAC) mergers instead of IPOs to go public in the United States. A total of 982 IPOs raised $308 billion in gross proceeds, an average of $314 million per deal, whereas 501 SPAC mergers raised $161 billion, with an average deal value of $322 million. Despite the importance of SPAC mergers as a major alternative to IPOs, academic understanding of the trade-off between the two is less developed.

The IPO process allows firms to learn about their quality via market demand (e.g. Ritter and Welch 2002; Hanley 1993), but limits investor access to the firm's future operational strategies due to the 'safe harbor' rule. In contrast, SPAC mergers are not bound by this rule, offering investors a more precise assessment of the firm albeit at a higher cost. Determining which of these forces predominates is an empirical question, which is hard to resolve using reduced-form techniques because the firm's decision-making is endogenous and there is no obvious instrument. Moreover, the answer also depends on unobservables, including the firm's quality and the precision and cost of investors' information production. To get at these unobservables and accurately measure the tradeoffs, I propose a tractable model where firms optimally choose between remaining private, IPO and SPAC merger, and empirically estimate it using a simulated method of moments (SMM) technique.

I find that early stage firms with more uncertainty about their qualities choose SPAC mergers. For these firms, the SPAC investor's precise evaluation substantially reduces the firm's uncertainty. Upon receiving positive feedback from the SPAC investor, these firms' equity value increases by 34.3% ($539 million), compared to the modest boost of 17.3%

[1] See Lucas and McDonald (1990); Zingales (1995); Black and Gilson (1998); Pagano, Panetta, and Zingales (1998); Chemmanur and Fulghieri (1999); Subrahmanyam and Titman (1999); Maksimovic and Pichler (2001); Schultz and Zaman (2001); Ritter and Welch (2002); Pástor and Veronesi (2005); Pástor, Taylor, and Veronesi (2009).

($256 million) from IPO investors. In contrast, high-quality firms prefer IPOs as the need for precise evaluations is less critical. The increase in equity value from the SPAC merger ($144 million, or 6.7%) does not significantly outweigh the increase from IPO ($83 million, or 4.1%).

In the proposed model, firms face uncertainty regarding their growth prospects, and learn about their quality over time. Moreover, as in the book-building literature, firms can learn from the financial markets as they go public. Here I assume that the firm can learn more in a SPAC merger because of the safe-harbor regulation. In addition, I assume the SPAC merger is more costly due to a more complex design. As a result, whether a firm should go public via IPO or SPAC depends on the marginal benefit from the extra information that the SPAC merger produces.

Firms make their choice of staying private, going public through IPO, or merging with SPACs by solving optimal timing problems. It is critical to specifically model firms' choice to stay private as the choice of going public is itself endogenous. In the model's economy, SPAC and IPO investors evaluate firms that attempt to go public. A negative evaluation results in the firm remaining private whereas a positive evaluation improves the belief of the firm's quality which increases the chance of a successful public issuance. Evaluation is costly and is compensated through equity payment, that is, the firm needs to leave more money to investors on a positive evaluation to compensate for the investor's loss on a negative evaluation. The model assumes that SPAC investors can produce a more precise evaluation, but also at a higher cost.

Firms choose the optimal way of going public based on prior beliefs of their quality, driven by positive and negative earnings shocks. In the model, the degree of uncertainty regarding the type of the firm is highest when the belief is in the middle range, that is, the firm has an equal chance of being high-quality or low-quality. These firms prefer SPAC mergers to IPOs because a more precise evaluation significantly reduces uncertainty and generates larger payoffs. When the uncertainty surrounding the firm's quality fades away, marginal benefits of a more precise evaluation are depressed. Therefore, firms with a stronger prior belief tend to opt for an IPO, while firms with a weaker prior belief tend to retain their private status.

To be more specific about the model, the economy consists of entrepreneurs who own private firms. These firms produce projects that are protected by patents. The nature of these projects varies, being either high-growth or low-growth. The entrepreneur is uncertain about the true quality of the firm. The firms continue their production activities, generating earnings until the expiration of the patent, which occurs exogenously. Here, the expiration of a patent can be interpreted as the entrepreneur's exit via mergers and acquisitions, or increased competition eventually drives the firm's abnormal returns away. Before the patent expires, all market participants symmetrically update their beliefs about the firm's quality.

Throughout a firm's lifetime, opportunities to go public arrive exogenously. These opportunities are essentially periods when market conditions are preferrable for firms to go public, as in Pástor and Veronesi (2005). When such an opportunity arrives, the entrepreneur has the choice to take the firm public by selling the firm to public investors. Part of the capital raised is immediately reinvested in the firm to expand the firm's operations.

Before investing, public investors independely evaluate the firm. This evaluation produces a binary signal, which updates the prior belief about the firm's quality. The evaluation is prone to type-II errors: while high-quality firms are identified accurately, errors may occur in evaluating low-quality firms. A positive signal increases the firm's prior belief, suggesting a higher expected growth rate. Conversely, a negative signal points to the firm undertaking a low-growth project. Successfully going public therefore offers two benefits: it provides extra capital for scaling up the firm's production and strengthens the belief of its quality. Together, these benefits contribute to a higher valuation of the firm's post-public equity.

However, the going public process is also costly. Investors need to pay a fixed cost for evaluating a firm. If the going public process is successful, this cost is offset through underpricing, where the firm intentionally sets a lower price for its shares, a practice referred to as leaving "money on the table" (MLOT). On the other hand, if the going public process fails, investors bear the cost of information production, which they incur beforehand. The model assumes that investors break even in expectation: for a successful public offering, the firm needs to underprice its shares enough to compensate for the potential risk of failure.

The entrepreneur has two options to take the firm public: IPO or SPAC merger.[2] The model makes a critical assumption on the difference between the two mechanisms: SPAC investors can evaluate the firm more accurately but at a higher cost. This assumption stems from the inherent institutional differences between the two methods: (1) Under the 'safe harbor' rule, SPAC investors have access to the firm's future business plans, a level of detail not permitted in the IPO process. This allows for more informed evaluations for SPAC investors. (2) The SPAC merger process is inherently more complex compared to the IPO process. The SPAC's founder and initial shareholders who take on the task of establishing the SPAC and transitioning it into a public company may ask for extra compensation. Empirical evidence also supports the above assumption. First, companies that go public through SPAC mergers on average leave $149 million 'on the table', significantly more than the $57 million left by firms undergoing IPOs. Second, the communication of future business plans is a key element in the interaction between the firm and SPAC investors. Utilizing a pre-trained Bidirectional Encoder Representations from Transformers (BERT) model, I analyze earnings call transcripts from the merger's announcement to its completion. These calls, similar in function to IPO roadshows, serve as a vital channel for investors to directly obtain information from the company. The analysis shows that approximately 10% of these discussions focus on the firm's specific future business strategies. This additional information could potentially help SPAC investors make more informed and accurate evaluations than IPO investors.

Building on the assumption, I solve for the private firm's optimal choice based on its

[2]For a more detailed comparison between SPAC and IPO, see Ritter, Gahng, and Zhang (Forthcoming).

current earnings and prior belief. Using a stochastic lumpy adjustments (SLAs) technique, similar to approaches in (Van Binsbergen and Opp, 2019; van Binsbergen and Opp, 2019; Opp, 2019), I derive a tractable global solution to the model. The solution identifies two prerequisites for a private firm to go public: (a) sufficient earnings and (b) a not-too-low prior belief. Adaquate earnings ensure that the benefits of scaling-up outweight the costs of investor evaluation. A not-too-low prior belief ensures that a positive evaluation can indicate a positive NPV for the firm. Without meeting these criteria, investors are likely to forego evaluation, resulting in the firm remaining private in equilibrium.

The model depicts a "U-shape" entry boundary for going public, based on the required levels of earnings and prior belief. Successful public entry not only expands the firm's production but also increases the firm's belief. The marginal benefit from this expansion increases with the firm's earnings, whereas the marginal increase in the firm's belief is concave in the firm's prior belief. To be more specific, firms with the highest uncertainty–those with roughly equal chances of being high-growth or low-growth–gain the most from an investor's positive evaluation. As a firm's prior belief starts from a lower point and increases, the marginal benefit of a positive evaluation increases, thereby reducing the earnings threshold needed for going public. This threhold reaches its lowest point when the firm's prior belief is moderate. As the prior belief continues to increase, the firm's confidence in being a high-growth type before any evaluation also rises, diminishing the incremental benefit of the evaluation. Consequently, the earnings threshold for going public tightens again, resulting in a "U-shape" entry boundary.

The model predicts a firm's optimal choice between an IPO and a SPAC merger based on the same logic. Firms with intermediate levels of prior belief, or those facing the greatest uncertainty, aim to generate as much information as possible. These firms favors SPAC merger as the precise evaluation significantly reduces the firm's uncertainty and improves the firm's post-public equity value that outweighs the higher costs. Conversely, firms with strong prior beliefs find less value in a more precise evaluation, as their uncertainty is already low. Thus, they tend to choose an IPO, which, despite offering a less precise signal, comes with lower costs.

One of the goals of this paper is to quantify the tradeoff presented in the model: specifically, the increase in a firm's equity value due to positive evaluations by SPAC and IPO investors, weighed against the associated costs. However, quantifying these tradeoffs poses a significant challenge. Firms' going public decisions are endogenous, which generates endogenous patterns in the firm's public offering outcomes. There are no obvious instruments. Additionally, several model elements are unobservable, such as the precision and costs of IPO and SPAC investors' evaluations. While reduced-form empirical techniques can indicate the direction of these tradeoffs, accurately measuring their magnitudes requires estimating an economic model.

These challenges naturally lead to a structural estimation approach. This approach infers unobservable variables from the observed endogenous patterns in firms' decisions to go public and the resulting outcomes. Employing a structural approach not only quantifies the magnitudes of these tradeoffs, but also allows us to explore counterfactuals, such as assessing the benefits and costs for a firm choosing a SPAC merger if it had instead opted for an IPO, and vice versa.

I estimate the model parameters using a simulated method of moments (SMM) strategy that utilizes data from firms that sought to go public between 2010 and mid-2023. The parameters I estimate encompass the growth rate for low-growth firms, the differential in growth rates between high and low-growth firms, the frequency of public offering opportunities, the rate of patent expiration (entrepreneur cash-outs), the precisions and costs of IPO and SPAC investors' evaluations, and the initial level of prior belief. The moments simulated from the model align closely with the empirical moments observed in the data, with no significant statistical discrepancies.

The model's estimates show that at the point of going public, the prior belief for IPO firms is approximately 0.90, indicating a 90 percent probability of being a high-growth firm. This belief is marginally increased to 0.91 following positive evaluations by the IPO investor, leading to a 4.1% raise in the firm's valuation, or $83 million. In return, the firm leaves the IPO investor $56 million, of which 93% ($52 million) covers the investor's actual cost and the remaining amount is risk markup. In contrast, SPAC investor evaluations play a more substantial role: firms opting for SPAC mergers have a lower prior belief of 0.72 at the time of going public. This belief is elevated to 0.78 after receiving positive evaluations, increasing the firm's equity value by 34.3%, or $539 million. In return, the firm leaves $158 million to investors, of which 76% covers the investor's actual cost and the remaining amount is risk markup.

These estimation results are also consistent with the observed characteristics of firms in the empirical data. Firms that merge with SPACs have fewer patents, smaller workforces, longer durations to secure initial financing, and a higher propensity to operate within the information technology sector. In addition, they need more help elevating their prior belief as they are backed by less reputable venture capitalists.

After estimating the model, I use it as a laboratory to quantify the actual tradeoffs between choosing an IPO and a SPAC merger. I do so by comparing simulated data between the estimated model and two counterfactual models. In the first counterfactual model, firms that initially choose a SPAC merger are simulated to opt for an IPO instead. I find that these firm's valuation would increase only by 17.3%, or $256 million, but at a cost of $60 million. This suggests that despite the higher costs, the benefits of a more precise evaluation outweight these expenses. In the second counterfactual model, firms that choose IPOs switch to SPAC mergers instead. The simulation results in a 6.7%, or $144 million, increase in their valuation, but incurs a substantial cost of $125 million.

Related Literature. My paper is related to the large IPO literature. Several factors can influence the timing and success of an IPO, including market conditions, the company's financial performance, and the regulatory environment. Studies have found that market conditions, such as the state of the stock market and the overall economic climate, can have a significant impact on the timing of IPOs (Ritter, 1987; Pástor and Veronesi, 2005). The financial performance of the company, such as its profitability and growth prospects, also plays a critical role in the success of an IPO (Welch, 1989; Megginson and Weiss, 1991). Finally, regulatory factors, such as changes to securities laws or accounting regulations, can also affect the IPO process and the cost of going public (Kim and Ritter, 1999). My paper contributes to the literature by looking how investors' evaluations on the firm's future growth can affect the entreprenur's going public decision.

My research contributes to a long line of research that studies institutional investors' role in producing information in the IPO process. Rock (1986) presents a model where some investors have information advantage over other investors and the firm needs to underprice its shares so uninformed investors participate in the issuing. Benveniste and Spindt (1989) predict positive average underpricing because investors are rewarded for value-relevant information. Yung (2005) models both the investment bank's screening and investor's evaluation. Empirical studies also provide evidence supporting public investors' information production rule, including (Bradley and Jordan, 2002; Lowry and Schwert, 2004). A recent study by Aryal et al. (2022) uses SPACs as an empirical setting and provide evidence on larger investors producing information in SPAC merger. My study contributes to their work by modeling the information production cost as a factor to consider when entrepreneur decides either to take the firm public.

I also contribute to a nascent literature that seeks to understand the role and functioning of SPACs. For instance, Ritter, Gahng, and Zhang (Forthcoming) study the cost and benefit of using SPACs to go public; Gofman and Yao (2022) study the conflict of interest and associated harm from directors serving on the boards of multiple SPACs; and Klausner, Ohlrogge, and Ruan (2022) analyze the structure of SPACs and the associated costs. Cumming, Haß, and Schweizer (2014) and Lakicevic and Vulanovic (2013) study the likelihood of

SPACs raising money. Fagan and Levmore (2022) and Levmore and Fagan (2022) study the role of PIPEs (Private Investor(s) in Public Equity) in the SPAC process. Dimitrova shows empirical evidence on implicit incentives in SPAC contracts may lead to bad SPAC performance. Kolb and Tykvova (2016) use data from 2003 to 2015 and find that small and levered firms with low growth opportunities tend to use SPAC to go public. Vulanovic (2017) study how institutional characteristics of SPACs are important in determining post-merger SPAC outcomes. Blankespoor et al. (2022) specifically look into financial projections made in the SPAC merger process and find that SPAC merger includes highly optimistic projections. In addition, there are several theoretical papers on SPACs. Gryglewicz, Hartman-Glaser, and Mayer (2021) model a financial sponsor's choice between starting private equity versus sponsoring a SPAC where adverse selection exists. Luo and Sun (2022) study the incentive misalignment between the SPAC sponsor and the SPAC investors. To the best of my knowledge, my study is the first one to model a firm's choice between IPO and SPAC merger, and uses structural estimation to directly quantify the tradeoff.

My paper relates to the extensive literature on how firms learn from the financial market. Firms can learn from prices when market participants have more information than the firm's management team (Hayek, 2013). Bond, Edmans, and Goldstein (2012) introduces a new definition of price informativeness as the degree to which prices reveal new information to managers. This concept is pivotal as firms incorporate this private information into their investment decisions (Chen, Goldstein, and Jiang, 2007; Foucault and Gehrig, 2008). Additionally, the regulatory environment, particularly regarding insider trading (Fernandes and Ferreira, 2009) and disclosure (Jayaraman and Wu, 2019; Blankespoor, deHaan, and Marinovic, 2020), plays a crucial role in determining the informativeness of prices, thereby influencing firms' investment decisions. Firms can also learn from prices of other firms in the same industry. Yan (2022) show that private firms tend to increase investment when public firms in the same industry are valued higher. Firms can also time their going public decisions based on industry dynamics (Lowry, 2003). My paper adds to this literature by studying which firms have the strongest incentive to learn from the market, and how this affects the firm's going public choice.

My paper contributes to the existing literature on investors learning about a firm's profitability and how this learning affects firms' IPO decisions. Pástor and Pietro (2003) provides a framework for evaluating a firm's market value in the presence of investor learning. Pástor and Veronesi (2005) argue that firms strategically time their IPOs based on favorable market conditions. Pástor, Taylor, and Veronesi (2009) focus on the entrepreneur's tradeoff between going public and staying private, showing that going public can be optimal when a firm's expected future profitability is high, as the market value of the firm exceeds the private value of the firm. My paper differs from previous work by examining the tradeoff between going public through merging with SPACs versus traditional IPOs.

For tractability, my model adopts a stochastic lumpy adjustments (SLAs) technique, which is also used in Van Binsbergen and Opp (2019); van Binsbergen and Opp (2019) and Opp (2019).

2 Data

2.1 Sources

To conduct the empirical analysis, this paper uses information on firms that were once private between January 2010 and June 2023. I compose a comprehensive data set that keeps track of these firm's status during the sample period. Specifically, the firm can be private throughout the sample period, exited through a merger and acquisition deal or liquidated, or went public through a traditional IPO process or a SPAC merger. The dataset provides detailed information on the firm's characteristics during the private stage, the public offering outcome, and the financial information after they went public. I introduce the different sources of data that I use below in detail.

Pitchbook database[3] provides financing information for these firms when they are still private. PitchBook collects detailed data on venture capital, private equity and M&A—including public and private companies, investors, funds, investments, exits and people. It is best known for venture capital (VC) and private equity (PE) data. Specifically, Pitchbook provides information including the firm's first invested financing capital, if the firm is backed by a venture capital (VC) investor, the firm's operated sector, the firm's employee history, whether the firm exited through a merger and acquisition deal, etc.

For the subset of firms that chose to go public through IPO, I get the IPO outcome from two data sources. The primary source of data on IPOs up to December 2022 is the SDC Platinum New Issues database[4]. For IPOs between January and June 2023, data was obtained from Stock Analysis[5]. The sample includes IPOs with an offer price of at least $5.00 but excludes ADRs, unit offers, closed-end funds, REITs, natural resource limited partnerships, small best efforts offers, banks and S&Ls, and stocks not listed on CRSP. CRSP includes stocks from Amex, NYSE, and NASDAQ.

For the subset of firms that chose to go public through the SPAC merger, I compile the SPAC merger outcome from two main sources: the SPAC Research Database[6] and the PrivateRaise's SPAC Search Database[7]. This combined dataset offers a comprehensive list of SPACs registered with the SEC from January 2010 to June 2023, detailing their ticker symbols, target companies, IPO dates, merger dates, and capital raised. Data accuracy was ensured by cross-referencing the datasets and removing duplicates.

Also for the subset of firms that chose SPAC, I manually collect their conference call transcripts that are related to the firm's roadshow to examine the communication between the firm and the SPAC investor. As the SPAC is publicly traded at the time of the roadshow, I find these transcripts in the firm's filings with the Securities and Exchange Commission

[3] https://pitchbook.com/
[4] https://www.refinitiv.com/en/products/sdc-platinum-financial-securities-data
[5] https://stockanalysis.com/
[6] https://www.spacresearch.com/
[7] https://www.privateraise.com/

(SEC), by looking at the 8-K statements around the time of the SPAC's merger announcement. I also collect the firm's earnings call transcripts during the SPAC merger process from the SEC's Edgar database.

In addition, post-public financial data for firms was derived from Compustat, with stock pricing data sourced from CRSP.

2.2 Sample construction

This study focuses on the firm's decision to go public versus staying private. The going public sample includes all firms that went public through either IPO or SPAC merger during the sample period of January 2010 to June 2023. The public sample contains 2,744 firms that went public during my sample period. Out of these firms, 802 firms tried to go public through merging with SPACs. Out of the 802 firms, 498 completed the merger process, 134 deals were broken, and 170 have announced targets but have not completed the merger process as of the end date of my sample. During the same sample period, 1,942 firms tried to go public through IPO. 1,737 firms successfully completed the IPO process, and the rest 205 firms eventually withdrew from the IPO process and went back to staying private. As of the second quarter of 2023, there are 215 active SPACs. These SPACs have collectively garnered $50.5 billion in proceeds, underscoring their substantial financial capability and emphasizing their role in aiding private firms to transition to public status via SPAC mergers.

2.3 Summary Statistics

In Table 1 (Panel A), I present statistics for 1,942 firms that transition to the public domain via IPOs. These firms have an average patent count of 0.03 at inception, which increases to a mean of 7.62 patents by the time they go public. They start with an average of 1,102 employees. Of these firms, 13% operate in the information technology sector, 28% are in healthcare, and the average VC reputation is 0.17, where the VC reputation is measured as the lead VC's proportion of total investment in the VC industry.

Table 1 (Panel B) delves into the data for 802 firms that choose the SPAC merger route. Compared to firms that went IPO, evaluating these firms both at their start and prior to going public can be more challenging. Their average initial patent count is close to zero, only increasing to 0.02 patents before their public listing. They typically commence with fewer employees, averaging 484. Within this group, 20% are associated with the technology sector, 15% belong to healthcare, and the average VC reputation is lower at 0.13.

The post-public equity value for the two sets of firms is comparable. SPAC firms grow at a lower rate than IPO firms and choose to access the public market earlier. In terms of the costs of public offerings, SPAC firms leave around 148 million dollars to investors (including all participants), measured by the difference in the value of purchased securities at the end of the merger and the offer price. In contrast, IPO firms leave only around 57 million dollars on the table (including underwriting fees), measured by the difference in the value of shares at the first-trading day and the offer price.[8]

2.4 Safe Harbor and Forward Looking Statements

One distinct feature of the SPAC merger is that SPAC investors can learn about the company's future business plan. This distinction arises from the "Safe Harbor" protections offered by the Private Securities Litigation Reform Act of 1995 (PSLRA), which I detail in Appendix A.2. In Appendix A.3, I give an example by comparing investor's presentations of Polestar which merged with SPAC, and Rivian which went IPO.

To understand if the SPAC investors utilize the safe harbor rule during the SPAC merger process, I examine the communication between the SPAC and investors. After the SPAC merger's announcement and before the merger is completed, the private company's top executives can present the company to investors during the SPAC's earnings conference

[8] Calculation of money left on the table for the SPAC merger is explained in Appendix A.1.

Table 1
Descriptive statistics

	Mean	Std.Dev.	p25	p50	p75	Obs.
A. IPO sample						
Firm Characteristics						
Patents at start	0.03	0.18	0.00	0.00	0.00	1,942
Patents pre-public	7.62	283.17	0.00	0.00	0.00	1,942
Employees at start	1,102	9,451	0	15	157	1,942
Years to first finance	5.92	16.36	0	0	3	1,942
VC reputation	0.17	0.38	0.00	0.00	0.17	1,942
Healthcare	0.28	0.45	0.00	0.00	1.00	1,942
IT	0.13	0.34	0.00	0.00	0.00	1,942
HHI	0.03	0.04	0.00	0.00	0.08	1,942
Public Offering Outcome						
MLOT	56.67	148.28	-0.22	10.83	59.24	1,382
Post-public valuation	2,100.68	4,098.73	281.04	697.12	1,913.85	1,211
Years to public	18.16	23.40	6.00	11.00	19.00	1,815
Growth rate	0.19	0.24	0.06	0.12	0.21	786
B. SPAC merger sample						
Firm Characteristics						
Patents at start	0.00	0.00	0.00	0.00	0.00	802
Patents pre-public	0.02	0.33	0.00	0.00	0.00	802
Employees at start	484	2,946	0	16	109	802
Years to first finance	6.64	16.63	0	1	5	802
VC reputation	0.13	0.40	0.00	0.00	0.03	802
Healthcare	0.15	0.36	0.00	0.00	0.00	802
IT	0.20	0.40	0.00	0.00	0.00	802
HHI	0.02	0.05	0.00	0.00	0.00	802
Public Offering Outcome						
MLOT	148.64	252.42	24.94	65.20	139.88	484
Post-public valuation	2,112.20	3,394.18	492.55	1,164.37	2,282.72	438
Years to public	16.94	21.69	6.00	10.00	18.00	595
Growth rate	0.15	0.18	0.04	0.09	0.18	287

This table presents descriptive statistics for firms in my sample. The sample covers the period from 2010 to mid-2023. All variables are winsorized at the 1st and 99th percentiles.

calls. These conferece calls play a similar role as the IPO's roadshow.

Compared to 10-K and news sources like the Wall Street Journal (WSJ), the signal-to-noise ratio in the earnings call is much stronger than most other corporate events (Garcia, Hu, and Rohrer, 2023). The core of my approach depends on the relationship between the information that investors get from the earnings call and the quality of the firm. Therefore, having a strong signal-to-noise ratio in my analysis is essential.

To pull out the word embeddings[9], I use the Bidirectional Encoder Representations from Transformers (BERT), an NLP algorithm developed by Google AI. BERT has some key advantages over other models. Unlike context-free models (e.g., Word2Vec, GloVE), BERT creates an embedding representation of a word based on its context (contextual representation). In contrast to unidirectional models (e.g., ELMo, ULMFit), which create a word's representation based on previous words, BERT is bidirectional, considering both preceding and subsequent context to form the embeddings of a word. Because of these features, BERT is very accurate in interpreting texts (Devlin et al., 2018) and has been used more in economic studies (Chava, Du, and Paradkar, 2020; Chava, Du, and Malakar, 2021; Gorodnichenko, Pham, and Talavera, 2023).

I use a pretrained BERT model, FinBERT (Araci, 2019), for my analysis, which can assign interpretations to texts, including sentiments (positive, neutral, or negative), and determine if a statement is forward-looking. FinBERT is trained on a large corpus of 4.9 billion tokens including corporate reports (10-K and 10-Q), earnings call transcripts, and analyst reports. Specifically, for sentiment analysis, I use the finbert-tone model, a version of FinBERT fine-tuned on 10,000 manually annotated (positive, negative, neutral) sentences from analyst reports. This model performs well on financial tone analysis tasks (Huang, Wang, and Yang, 2023). In addition, I identify forward-looking statements from the SPAC firm's earnings calls at the merger announcement using FinBERT-FLS, a FinBERT model fine-tuned on 3,500 manually annotated sentences from the Management Discussion and Analysis section of annual reports from Russell 3000 firms.

I use the pretrained FinBERT model to analyze the text of the earnings transcripts

[9]In simple terms, word embeddings are mappings of words to vectors of real numbers, capturing the meanings of the words and the grammatical relationships between them.

Table 2
Summary of Forward-looking Statements in SPAC Conference Call

	Obs.	Mean	Std	P10	P50	P90
Sentence Count	709	183.10	121.68	40.00	167.00	371.20
Forward-Looking Classification						
Forward-looking sentences (FLS)	709	22.17	18.27	0.00	18.00	48.20
Specific FLS	709	15.96	13.75	2.00	12.00	35.00
Non-specific FLS	709	6.73	6.57	0.00	5.00	16.00
% Forward-looking sentences (FLS)	709	12.04	8.35	0.00	10.96	22.59
% Specific FLS	709	9.71	7.52	2.36	7.73	19.67
% Non-specific FLS	709	3.52	2.62	0.00	3.14	6.72

This table provides descriptive statistics for the earnings press release sentence types for the sample of SPAC firms at the merger announcement.

on a sentence-by-sentence basis. After removing boilerplate language, such as that attributable to Forward-looking Statements, I classify sentences in an earnings transcript as specific forward-looking sentences (Specific-FLS)[10], non-specific forward-looking sentences (Non-Specific FLS)[11], and not foward-looking sentences (Not-FLS).

There are 279 SPACs that filed earnings transcripts with the SEC at the time of their merger announcement, with a total number of 709 transcript files. Table 2, Panel A provides descriptive statistics for the earnings announcements in my sample for SPAC firms at the time of the merger announcement. Earnings announcements have an average of 183.10 sentences (after removing boilerplate language and sentences with less than 10 words), of which 22.17 (12.04%) are classified as forward-looking. Of those forward-looking statements, an average of 15.96 (9.71%) statements are classified as statements that relate to the forecast of specific information (specific FLS), and 6.73 (3.52%) are classified as general forecasts (non-specific FLS).[12]

[10]An example of the specific forward-looking sentence is "We look forward to providing more detailed information regarding our strategic plans, important milestones and our progress in the coming quarters for both our Gen 4 vehicles and our Gen 5 ground-up chassis vehicles, which will follow subsequently behind Gen 4."

[11]An example of the non-specific forward-looking sentence is "So we're in the early stages of that, but we're seeing benefits already. And we think as we continue to innovate and create new opportunities to expand with those customers, that strategy is going to pay off."

[12]Besides future production plans, firms can also make financial projections. Blankespoor et al. (2022) show that the projections are overly optimistic, leaving a lot of room for the investor's own evaluation.

Overall, the data indicate three things about firms that chose to merge with SPAC: (1) when private firms present to investors in SPACs' earnings calls, a significant fraction of the communication is forward-looking statements that are specific about the private firm's future plans. (2) Compared to firms chose IPO, firms choosing SPAC mergers face a harder time resolving their uncertainty when remaining private as they have less patents, less employees, operate more in the IT sector, and backed by less reputable VCs. (3) Compared to the IPO process, firms merging with SPACs leave more money on the table to market participants. Motivated by these empirical findings, I build a model on the firm's endogenous choice of the optimal way of going public in the next section.

3 Model

I build the model using an SLA technology specification consistent with the principles laid out in Van Binsbergen and Opp (2019), van Binsbergen and Opp (2019), and Opp (2019). I make three specific modeling choices that, together, will ensure that solving the model involves only inverting sparse matrices.

There are two classes of risk-neutral agents, investors and an entrepreneur, who have identical preferences. I start by describing the firm's earning process.

3.1 Environment

At time $t = 0$, the entrepreneur is endowed with a private firm implementing a patent-protected technology. There are two possible outcomes of implementing the technology. First, the technology may generate a positive net present value (NPV) project, in which case the firm is denoted as high type ($\theta = h$). Second, the technology can also generate a negative NPV project, in which case the firm is denoted as low type ($\theta = l$). The firm's type is not directly observable. At date $t = 0$, nature determines the value of θ, where h is drawn with probability π_0, and l is drawn with complementary probability $1 - \pi_0$.

By implementing the technology, the firm makes earnings at each time, denoted by B_t. The firm's log-earnings process b_t is governed by a stochastic lumpy adjustment (SLA) process with both positive and negative Poisson shocks.

The arrival intensities of shocks to the state b are affected by the firm's type. Let $\lambda_b^+(\theta)$ and $\lambda_b^-(\theta)$ denote the Poisson arrival rates of upward and downward shocks to b, the distribution of the Poisson process can be written as:

$$N_{b,t}^+(\theta) \sim \text{Poisson}(\lambda_b^+(\theta)),$$
$$N_{b,t}^-(\theta) \sim \text{Poisson}(\lambda_b^-(\theta)),$$

where $N_{b,t}^+(\theta)$ and $N_{b,t}^-(\theta)$ denote the Poisson processes of positive and negative earnings shocks within time t for a firm of type θ.

When a positive shock hits, the firm's log-earnings increase by a fixed amount Δ_b, and when a negative shock hits, the firm's log-earnings decrease by the same amount. The dynamics of the firm's log-earnings process can be written as:

$$db_t = \Delta_b \cdot \left[dN_{b,t}^+ - dN_{b,t}^- \right]. \tag{1}$$

Given these dynamics, b_t takes values in a discrete set Ω_b, the elements of which constitute a grid with increments of size $\Delta_b > 0$, and the earnings B_t then follows the jump-diffusion process:

$$\frac{dB_t}{B_t} = \left(e^{\Delta b} - 1 \right) dN_{b,t}^+ + \left(e^{-\Delta b} - 1 \right) dN_{b,t}^-. \tag{2}$$

The firm keeps producing earnings until its patent expires at some future time T. The patent expiration date arrives at hazard rate η, i.e., the firm's patent in expectation lasts for

$\frac{1}{\eta}$. The firm is assumed to pay no dividends and reinvest all of its earnings, and to be financed only by equity. Note that since b_t follows a random process, B_t also evolves randomly, so B_T is unknown before time T.

I assume that the volatility of b does not contain incremental information about the hidden state θ. This is the case if the total Poisson arrival rate with which any adjustment to b occurs does not depend on the hidden state θ, although it may vary with any observable state or firm characteristic. Specifically, the total jump intensity is given by:

$$\lambda_b \equiv \lambda_b^+(\theta) + \lambda_b^-(\theta) \quad \text{for all } \theta. \tag{3}$$

This specification implies that the mean and the volatility of adjustments to b are given by:

$$\mu_b(\theta) = \Delta_b \left(\lambda_b^+(\theta) - \lambda_b^-(\theta) \right), \tag{4}$$

$$\sigma_b = \Delta_b \sqrt{\lambda_b}. \tag{5}$$

That is, as intended, the drift μ_b depends on the hidden state θ, whereas the volatility σ_b does not. As in models where earnings follow a Brownian motion with hidden drift, the dynamics for b are fully described by the drifts $\mu_b(\theta)$ and the volatilities σ_b; given these objects, the Poisson intensities $\lambda_b^+(\theta)$ and $\lambda_b^-(\theta)$ are uniquely pinned down, given any choice for Δ_b. Correspondingly, I will calibrate the model by directly choosing values for the drifts $\mu_b(\theta)$ and volatilities σ_b, and back out the associated Poisson intensities. Crucially, the second modeling choice will imply that beliefs about the state θ exhibit stochastic lumpy adjustments after earnings shocks are realized and stay constant otherwise.

Prior to the patent's expiration, opportunities to go public emerge at a Poisson rate of ρ. Upon the arrival of such an opportunity, the entrepreneur must choose between taking the company public or maintaining its private status. To enter the public market, the entrepreneur has the option to either undergo the conventional IPO process or merge with a SPAC. In the next subsection, I explain the going public process in detail.

Now consider a firm whose current log-earnings is b_t, and it is considering going public through the traditional IPO process.

The IPO process is costly. To get through the IPO process, the firm needs to pay fees to the underwriter and to the investor to prepare all the documents necessary and to conduct an evaluation of the firm, where the total cost is denoted by c_{IPO}. The evaluation process generates an independent signal of the firm's quality. The signal generated by the investor[13] can be either good ($s = H$) or bad ($s = L$), and the signal is independent of the prior information and satisfies

$$\Pr(s = L|\theta = l) = q_{\text{IPO}}, \tag{6}$$

$$\Pr(s = H|\theta = h) = 1. \tag{7}$$

Note that the investor's signal only admits type II errors. By Bayes' rule, given a bad signal, the firm is of low type with probability 1; given a good signal, the firm may be of high or low type. I verify the Martingale condition in Appendix A.1.

Using the newly generated signal, participants in the market update their beliefs about the quality of the firm. If the updated belief indicates that the firm is a positive NPV project, the firm signs a sales contract with the investor by selling the firm at price p to the investor, and reinvests part of the proceeds back into the firm's production.

Alternatively, the firm can choose to sell the firm to SPAC investors through merging with a SPAC. It is largely similar to the IPO process, the only difference is that SPAC investors get a signal with precision q_{SPAC} at a cost c_{SPAC}.

[13] In reality, there are multiple investors. Here one can think all other investors delegate the information production task to one investor.

3.2 Analysis

In this section, the equilibrium dynamics of a perfect Bayesian Nash equilibrium are characterized, with a focus on the information updating process.

3.3 Information Updating

Both investors and the entrepreneur possess identical priors concerning the firm-specific hidden state, denoted as θ. They gain insights from the realized earnings, with all other parameters being known. At time t, the agent's belief that the firm is in a good state is represented by π_t, which can be expressed as $\pi_t \equiv \Pr[\theta = h|\mathcal{F}_t]$. Here, $\mathcal{F}_t$ refers to all historical information accessible up to time t.

Understanding a firm's type is essential when determining upscale production strategies. When a firm has a project with a positive Net Present Value (NPV), there's a clear inclination to upscale. Conversely, if the project has a negative NPV, the firm might be hesitant.

Firms can determine their type through two main methods. One method is to remain private and continue production. Over time, the earnings from this production provide insights, updating the firm's understanding of its type. The alternative is to go public, allowing firms to benefit from the insights provided by public investors' signals.

However, transitioning to public status is not without costs. Firms need to consider the trade-offs, balancing the benefits derived from public signals with the associated costs. Interestingly, firms with different prior beliefs may benefit differently from the signal. The subsequent lemma delves into how a firm's prior belief interacts with a received high signal.

Lemma 1 (Posterior increase with high signal)**.** *Let a firm's posterior belief, after receiving a high-type signal, be denoted as $\pi(\theta = h|s = H, \mathcal{F}_t)$, its prior belief be denoted as $\pi(\theta = h|\mathcal{F}_t)$, and the precision of the signal be denoted as q. We have,*

1. *The firm's posterior belief increases upon receipt of a high-type signal, i.e., $\pi(\theta =$*

$h|s = H, \mathcal{F}_t) > \pi(\theta = h|\mathcal{F}_t)$.

2. The magnitude of the increase in the posterior belief, following the high-type signal, is a function of the firm's prior belief. Specifically, as $\pi(\theta = h|\mathcal{F}_t)$ increases from a low value, the magnitude of the increase in the posterior first rises and then falls.

3. The prior belief, $\pi^*(\theta = h|\mathcal{F}_t)$, that has the maximum of the increase in the posterior, diminishes as the signal's precision, q, rises. That is, $\frac{\partial \pi^*(\theta=h|\mathcal{F}_t)}{\partial q} < 0$.

Proof. See Appendix A.2. □

The first point of Lemma 1 shows that all firms benefit from a high signal in terms of increasing the belief of being a positive NPV project.

The second point of Lemma 1 illustrates that the marginal value of information is highest when the belief about the firm's quality is in an intermediate range. Given a low prior belief, the company is almost certain that it is a low-growth type. The perspective it holds is too rooted in pessimism, and it may receive little increase in optimism with a positive signal from public investors. Conversely, while possessing a strong prior belief, the firm has a high level of confidence that it is a high-growth type. Receiving additional positive news only serve as a confirmation of its performance and does not significantly update the belief. When the prior belief is in the middle range, the uncertainty about the firm's type is the highest. In this range, any substantial information can serve as a powerful factor in determining the firm's value.

In Lemma 1, the third point shows a clear relationship: the value a firm gets from a high signal is tied to how precise that signal is. Put simply, the more precise the signal, the lower the prior belief of the firm that benefits the most from it. If we consider a situation where SPAC investors offer more accurate signals than IPO investors, this would mean that firms opting for SPACs typically have a lower prior belief.

Next, I detail the process of information updating with respect to earnings shocks and any public signal generated. Agents update their beliefs based on Bayesian rules when they observe shocks to log-earnings b_t before and after the firm goes public. A positive shock to b increases the log-odds ratio by the log-Bayes factor f^+, while a negative shock decreases it by the log-Bayes factor f^-. The log-Bayes factors are determined by the grid increments Δ_b, the drifts $\mu_b(\theta)$, and the volatility σ_b. As the total arrival intensity λ_b is independent of the hidden state θ, the time elapsed between shocks to b does not provide agents with extra information. Agents also update their beliefs based on Bayesian rules when they observe the signal generated by public investors.

The following lemma characterizes the evolution of these beliefs:

Lemma 2 (Bayesian updating)**.** *The log-odds ratio $o_t \equiv \log[\pi_t/(1-\pi_t)]$ evolves according to the following process:*

$$\begin{aligned} do_t &= f^+ dN^+_{b,t} - f^- dN^-_{b,t} \\ &+ [(o_t + f_{IPO})\mathbb{1}_{s=H} - \infty \mathbb{1}_{s=L} - o_t]\, dN_{IPO,t} \\ &+ [(o_t + f_{SPAC})\mathbb{1}_{s=H} - \infty \mathbb{1}_{s=L} - o_t]\, dN_{SPAC,t} \end{aligned} \tag{8}$$

where I define the log-Bayes factors associated with positive and negative shocks to b:

$$f^+ \equiv \log\left[\frac{\lambda_b^+(h)}{\lambda_b^+(l)}\right] = \log\left[\frac{1+\Delta_b \frac{\mu_b(h)}{\sigma_b^2}}{1+\Delta_b \frac{\mu_b(l)}{\sigma_b^2}}\right], \tag{9}$$

$$f^- \equiv -\log\left[\frac{\lambda_b^-(h)}{\lambda_b^-(l)}\right] = -\log\left[\frac{1-\Delta_b \frac{\mu_b(h)}{\sigma_b^2}}{1-\Delta_b \frac{\mu_b(l)}{\sigma_b^2}}\right], \tag{10}$$

and the increase in the log-odds ratio associated with a good signal generated by public investors:

$$f_{IPO} = \log\left[\frac{1}{1-q_{IPO}}\right], \tag{11}$$

$$f_{SPAC} = \log\left[\frac{1}{1-q_{SPAC}}\right]. \tag{12}$$

Proof. See Appendix A.4. □

To ensure the log-odds ratio only assumes values on a grid with increments of size $\Delta_o > 0$, a mild parameter restriction is imposed. This constraint is the third critical modeling choice for maintaining the model's tractability, enabling the problem of optimal default to be solved using computationally efficient methods.

Assumption 1. *Parameters adhere to the restriction that* $\frac{f^+}{\Delta_o}$, $\frac{f^-}{\Delta_o}$, $\frac{f_{IPO}}{\Delta_o}$, $\frac{f_{SPAC}}{\Delta_o}$ *are natural numbers.*

While this constraint is vital for ensuring the model's tractability in the presence of learning, it is important to note that it imposes virtually no limitations on the parameter choices determining profitability dynamics, as Δ_o can be set to an arbitrarily small value. By enforcing this constraint, precise global solutions can be obtained, requiring only matrix inversions.

3.4 Post-public Equity Value

Employing the principle of backward induction, we initiate the analysis of the company's equity value after its transition to public status. The state vector is symbolized by (b, o). $n = n_b \times n_o$ is determined as the cumulative quantity of discrete states. The entrepreneur's value function can be denoted by $V_{post}(b, o)$, exhibiting the Hamilton-Jacobi-Bellman (HJB) equations in the succeeding form:

$$\begin{aligned} 0 = & -rV_{post}(b,o) + \eta(e^b - V_{post}(b,o)) \\ & + \mathbb{E}\left[\lambda_b^+|o\right]\left[V_{post}(b+\Delta_b, o+f^+) - V_{post}(b,o)\right] \\ & + \mathbb{E}\left[\lambda_b^-|o\right]\left[V_{post}(b-\Delta_b, o-f^-) - V_{post}(b,o)\right], \end{aligned} \tag{13}$$

where

$$\mathbb{E}\left[\lambda_b^+|o\right] = \pi(o)\lambda_b^+(h) + (1-\pi(o))\lambda_b^+(l)$$
$$\mathbb{E}\left[\lambda_b^-|o\right] = \pi(o)\lambda_b^-(h) + (1-\pi(o))\lambda_b^-(l).$$

3.5 Equity Value of Going Public

When the firm goes public, it offers a sales contract to the public investor, ensuring that the investor breaks even. The investor then receives the proceeds net of the investment, leading to the following system of equations in the case of an IPO:

$$\Pr(s=H)(V_{\text{Post}}(b+k\Delta b, o+f_{\text{IPO}}) - p_{\text{IPO}}) = c_{\text{IPO}}, \tag{14}$$
$$V_{\text{IPO}}(b,o) = \Pr(s=H)(p_{\text{IPO}} - (e^{k\Delta b}-1)e^b) + \Pr(s=L)V_{\text{Post}}(b,-\infty), \tag{15}$$

where Equation (14) articulates that the investor and the underwriter break even ex-ante in expectation, and Equation (15) specifies the total expected equity value for the entrepreneur if the entrepreneur opts to go public through the traditional IPO process.

By substituting $\Pr(s=H) = \pi_t + (1-\pi_t)(1-q_{\text{IPO}}) = 1 - \frac{q_{\text{IPO}}}{1+e^{o_t}}$ into Equation (14), we can derive an expression for the selling price p_{IPO}:

$$p_{\text{IPO}} = V_{\text{Post}}(b+k\Delta b, o+f_{\text{IPO}}) - \frac{c_{\text{IPO}}}{1-\frac{q_{\text{IPO}}}{1+e^{o_t}}}, \tag{16}$$

Equation (16) explicates the selling price of the firm, composed of two components. The first component represents the firm's post-merger equity value after considering the newly raised investment and after a positive signal from the IPO investors. The second component refers to the total amount of money left on the table ("MLOT"), i.e., the total dollar amount from underpricing that the firm needs to leave to the underwriter and investor during the

IPO process.

If the information production cost is higher, signifying that c_{IPO} is greater, the firm must underprice its shares more substantially. Furthermore, if the IPO investor's signal is more precise, denoted by a higher q_{IPO}, the underpricing degree will also be larger. However, a more accurate signal also amplifies the increase in the firm's log-odds ratios, f_{IPO}, thus enhancing the post-merger valuation. This means that the firm will likely be a more positive NPV project upon receiving a favorable signal.

Overall, the firm confronts a tradeoff from a more precise signal in two significant ways: it heightens the value of a positive signal but also necessitates that the firm compensate the investor by leaving additional money on the table. By substituting Equation (16) into Equation (15), we derive a formal expression for the firm's equity value when going IPO, as follows:

$$V_{\text{IPO}}(b,o) = (1 - \frac{q_{\text{IPO}}}{1+e^o}) \left[V_{\text{Post}}(b + k\Delta b, o + \log[\frac{1}{q_{\text{IPO}}}]) - \frac{c_{\text{IPO}}}{1 - \frac{q_{\text{IPO}}}{1+e^{o_t}}} - (e^{k\Delta b} - 1)e^b \right] + \frac{q_{\text{IPO}}}{1+e^o} V_{\text{Post}}(b, -\infty), \tag{17}$$

Similarly, we can derive the equation for the firm's choice to go public via a SPAC merger as:

$$V_{\text{SPAC}}(b,o) = (1 - \frac{q_{\text{SPAC}}}{1+e^o}) \left[V_{\text{Post}}(b + k\Delta b, o + \log[\frac{1}{q_{\text{SPAC}}}]) - \frac{c_{\text{SPAC}}}{1 - \frac{q_{\text{SPAC}}}{1+e^{o}}} - (e^{k\Delta b} - 1)e^b \right] + \frac{q_{\text{SPAC}}}{1+e^o} V_{\text{Post}}(b, -\infty), \tag{18}$$

The break-even function for SPAC investors can be derived too as follows,

$$p_{SPAC} = V_{\text{Post}}(b + k\Delta b, o + f_{\text{SPAC}}) - \frac{c_{\text{SPAC}}}{1 - \frac{q_{\text{SPAC}}}{1+e^{o_t}}}, \tag{19}$$

3.6 Firm's Choice

We can now describe the firm's decision-making process among staying private, going public through IPO, and going public through a SPAC merger in the following HJB equation:

$$
\begin{aligned}
0 = &- rV(b,o) + \rho\left[\max\{V_{\text{IPO}}(b,o), V_{\text{SPAC}}(b,o), V(b,o)\} - V(b,o)\right] + \eta(e^b - V(b,o)) \\
&+ \mathbb{E}\left[\lambda_b^+ | o\right]\left[V(b+\Delta_b, o+f^+) - V(b,o)\right] \\
&+ \mathbb{E}\left[\lambda_b^- | o\right]\left[V(b-\Delta_b, o-f^-) - V(b,o)\right],
\end{aligned} \tag{20}
$$

where V_{IPO} and V_{SPAC} are defined in Equations (17) and (18), respectively.

In Equation (20), $V(b,o)$ represents the expected value from keeping the firm private. If this value is less than the expected value of taking the firm public through either IPO ($V_{\text{IPO}}(b,o)$) or SPAC merger ($V_{\text{SPAC}}(b,o)$), the firm will elect to go public. This decision-making process encapsulates the trade-offs and strategic considerations involved in determining the optimal path for the firm.

4 Model Solution and Predictions

The firm's earnings and posterior belief about the firm's type fluctuate over time as earnings shocks arrive. The firm goes public as soon as the earnings and the posterior belief pass over an endogenous threshold. The threshold depends on all model parameters. In addition, the thresholds for IPO and SPAC can be different.

To analyze the firm's optimal decisions of going public, I solve the model numerically, using the parameters from Section 5. Figure 1 plots the pairs of probability belief (π_t) and the firm's earnings (B_t) for which the firm optimally decides to stay private, go public through SPAC, or go public through IPO.

Firms go public through SPAC when B_t and π_t lie inside the "entry region" in the

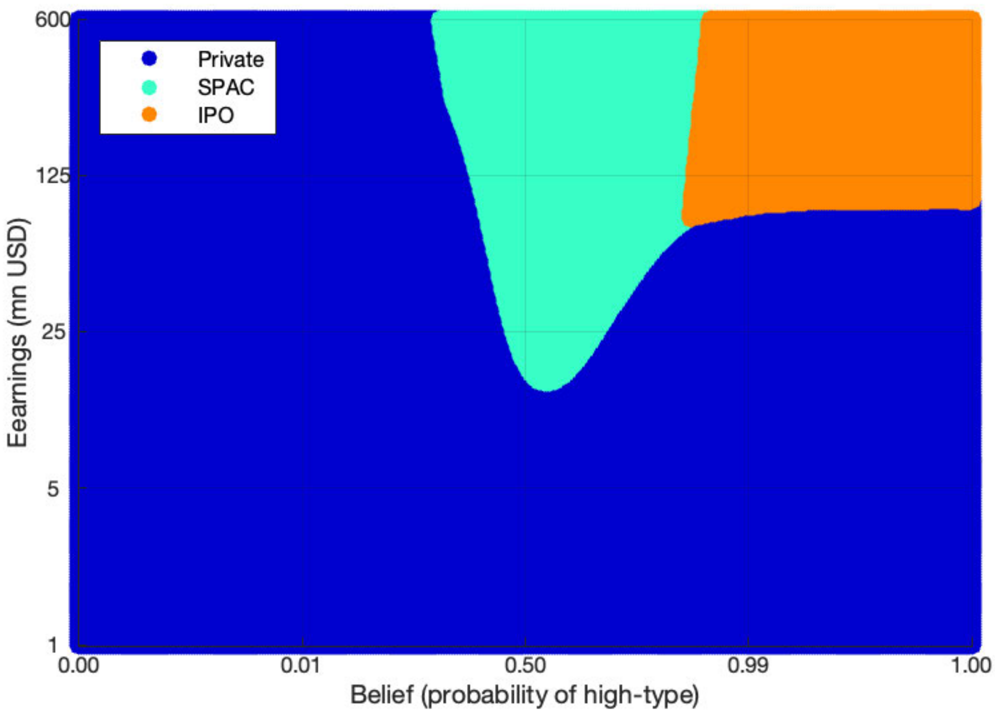

Figure 1: Going public decision.

This figure plots the firm's optimal decision; that is, the set of pairs of probability belief (horizontal axis) and earnings (vertical axis) that is optimal for the firm to stay private (blue), going public through SPAC merger (green), and going public through IPO (orange).

middle green area, and go public through IPO when B_t and π_t lie inside the entry region in the northeast orange area. If the firm is endowed with a project with B_t and π_t inside either of the two entry regions, the firm goes public immediately, through SPAC and IPO accordingly. Otherwise, the firm stays private until it gets more mature and goes public as soon as either of the two entry boundaries is reached. If neither of the two boundaries is reached before the patent expires, the firm never goes public.

First, consider the tradeoff between staying private and going public. From the entrepreneur's point of view, a successful going-public process allows the entrepreneur to cash out by selling the firm to the investor. Using the money raised, the firm gets scaled up by a factor of K. In addition, the firm's posterior belief increases. Both factors increase the firm's post-public equity value. After the purchase, the investor gets the firm's future cash flows. Thus, the starting point of successful public selling is that the firm's posterior NPV needs

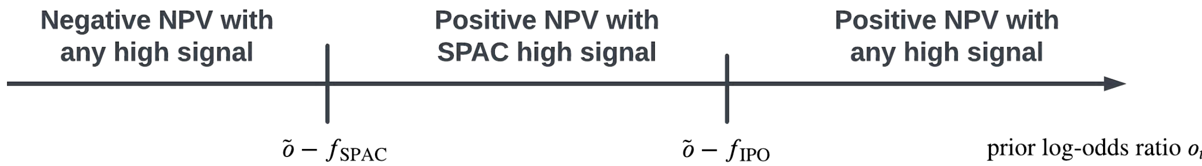

Figure 2: Evaluation result given a positive signal.

This figure plots the evaluation result given a positive signal from the public investor based on the firm's prior belief. $\tilde{o}$ is the threshold log-odds ratio that makes the firm's posterior project with zero NPV. f_{IPO} is the amount of increase in the firm's log-odds ratio with a positive IPO signal, and f_{SPAC} is the amount of increase in the firm's log-odds ratio with a positive SPAC signal.

to be positive, given a positive public signal. Otherwise, the investor will not be willing to make the purchase. I now calculate the firm's posterior NPV below,

$$\text{Firm's posterior NPV}|_{s=H} = V_{\text{Post}}(b + k\Delta b, o + f_{\text{SPAC/IPO}}) - Ke^b,$$

in Proof A.3, I show that the firm's posterior value is scalable in the firm's earnings level, i.e., $V_{\text{Post}}(b, o) = e^b \tilde{V}_{\text{Post}}$. I can rewrite the equation above as,

$$\text{Firm's posterior NPV}|_{s=H} = Ke^b(\tilde{V}_{\text{Post}}(o + f_{\text{SPAC/IPO}}) - 1),$$

thus, for a private firm to have a positive NPV after the public selling, the condition $\tilde{V}_{\text{Post}}(o + f_{\text{SPAC/IPO}}) \geqslant 1$ needs to be satisfied. There exists a threshold log-odds ratio, $\tilde{o}$ such that $\tilde{V}_{\text{Post}}(\tilde{o}) = 1$, and since $\tilde{V}_{\text{Post}}(o)$ is an increasing function in o, I have the following lemma,

Lemma 3. *Private firms that have a prior belief below $\tilde{o} - f_{SPAC}$ will be evaluated as a negative NPV project with any positive signal from public investors. Private firms that have a prior belief within $(\tilde{o} - f_{SPAC}, \tilde{o} - f_{IPO})$ will be evaluated as a positive NPV project upon receiving a positive signal from SPAC investors, but a negative NPV project upon receiving a positive signal from IPO investors. Private firms that have a prior belief above $\tilde{o} - f_{IPO}$ will be evaluated as a positive NPV project upon receiving either a positive SPAC or IPO signal.*

Figure 2 visualizes the evaluation result illustrated in Lemma 3. Lemma 3 predicts a firm's optimal decision based on their prior belief: for firms that are currently most likely a low-type firm, the project will be evaluated as a negative NPV project even with positive signals from public investors. For such firms, the investor will not be willing to make the purchase, and thus in equilibrium, these firms stay private. For firms that have a prior belief in the middle area, these firms face the highest uncertainty with regard to their true type. For these firms, if the SPAC investor gives a high signal, they will be evaluated as a positive NPV project, but will still be evaluated as a negative NPV project upon receiving a high signal from the IPO investor. These firms benefit from a more precise signal from SPAC investors the most: they will not have the opportunity to access the public market without SPAC. Lastly, for firms that have a high prior belief, i.e., they are more likely a high-type firm, they will be evaluated as a positive NPV project with either a SPAC or an IPO high signal. For these firms, it is natural that they do not feel the need to pay an extra cost to go SPAC in exchange for a more precise signal. If possible, they will choose to access the public market through IPO.

The solution in Figure 1 aligns with Lemma 3's predictions. In equilibrium, firms remain private when their prior belief is low, which represents the left blue region. Firms that face the highest uncertainty choose to access the public market through SPAC, which corresponds to the green region in the middle around the prior belief of 0.5. Firms that are high in prior belief choose to go public through IPO, in line with the orange region in the northeast region.

Lemma 3 represents the scenario of first best: positive NPV projects go public and scale up, and negative NPV projects stay private. However, the firm's ultimate decision to go public also depends on the relative cost of going IPO and SPAC. Specifically, the cost comes from the investor's effort to evaluate the firm. With such evaluation cost, some positive NPV firms may still choose to stay private, as the benefit of scaling up may be relatively small after compensating the investor by leaving a lot of money on the table.

The benefit of going public lies in the scaling-up effect: using the money raised in the public offering, the firm reinvests part of the money back into production, and scales up the firm's production by a fixed factor K. Thus, this scaling-up effect is larger when the firm is larger in size, i.e., b_t is larger. If we fix the firm's prior belief at constant, as we increase the firm's earnings level, the firm's value of going public becomes larger and eventually, it is optimal for the firm to access the public market. This aligns with the fact that small firms in equilibrium stay private, corresponding to the bottom blue regions in Figure 1.

Going public successfully not only scales up the firm's production but also increases the firm's posterior belief, both of which pass through to the firm's post-public valuation, which can be seen from $V_{\text{Post}}(b_t + k\Delta b, o_t + f_{\text{SPAC/IPO}}) = Ke^b\tilde{V}_{\text{Post}}(o_t + f_{\text{SPAC/IPO}})$. On the other hand, if the firm chooses to stay private, other than an option value of going public in the future, the firm's valuation can be approximated by $V_{\text{Post}}(b_t, o_t) = e^b\tilde{V}_{\text{Post}}(o_t)$. Thus, the marginal benefit from going public versus staying private depends on both the firm's size and the increase in the firm's unit post-public value $\tilde{V}_{\text{Post}}$ with a positive SPAC or IPO signal.

Lemma 1 shows that firms that face the highest uncertainty with regards to its type, i.e., firms with prior belief in the middle, have the largest increase in their posterior beliefs in response to a positive signal. Correspondingly, the unit post-public equity value also increases the most when the firm's prior belief is in the middle. Consider the following dynamics: a firm's prior belief increases from the $\tilde{o} - f_{SPAC}$ in Figure 2. As the prior belief approaches the middle, or equivalently, as the firm's uncertainty with respect to its type becomes larger, the benefit from going public in terms of the increase in the firm's unit post-public value becomes larger, thus, it allows firms of smaller size to access the public market. This explains the negative slope of the entry boundary on the SPAC (green) area on the left in Figure 1. Now, if we keep increasing the firm's prior belief after a cutoff point, the benefit from the increase in the unit shrinks. As a result, the entry boundary requires the firm to be larger in size. To better understand this point, Figure 3 plots the firm's post-public equity value at different values of size and belief. It shows that the firm's post-public value has the sharpest increase around the middle area of belief.

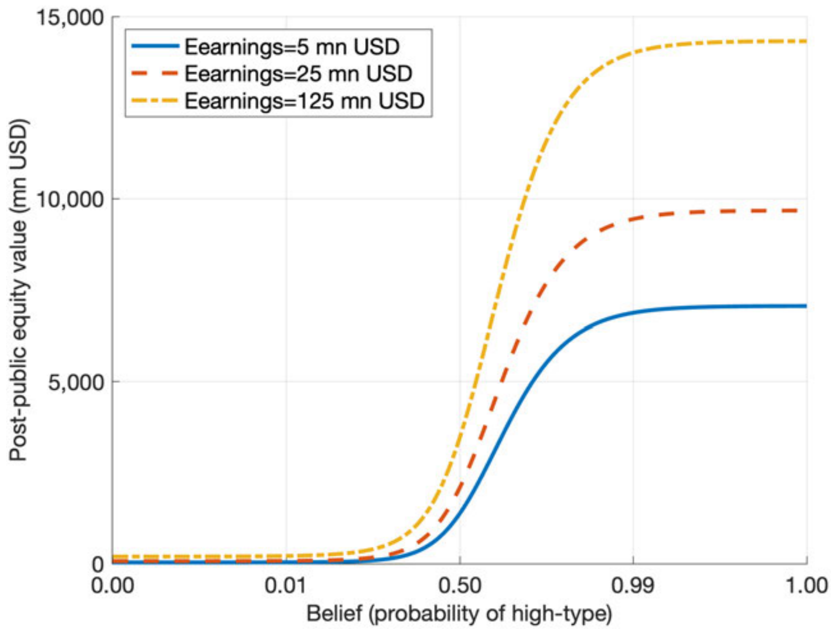

Figure 3: Post-public equity value.
This figure plots the firm's post-public equity value as a function of the firm's belief, at three different earnings levels.

In terms of economic meanings, firms with prior beliefs in the middle are the set of firms that face the highest uncertainty about their growth. Investor's signal will decrease their uncertainty by the largest amount, as a result, their marginal benefit from the investor's evaluation is the highest. Thus, these firms are willing to access the public market at a smaller size when the scaling-up effect is smaller.

5 Estimation

I structurally estimate most of the parameters of my model via a simulated minimum distance estimator discussed below. Before estimating the model, I set the value of the constant interest rate parameter, r, to 0.05, leading to a discount factor of 0.95. This value aligns with the typical annual discount factors observed among investors.

I also estimate several parameters separately. To start with, I estimate the firm's initial earnings level, B_0, using the firm's capital raised in its first financing. In addition, I separately estimate the level for technology firms and non-technology firms, to account for the different capital raised levels in the data for these two subsets of firms.

Next, I estimate the scaling factor K. By definition, K is the ratio of capital after and before the firm goes public. I use the firm's asset level after its going public process and the difference between the firm's asset level and the capital raised in the public process through issuing primary shares to estimate K.

Next, I estimate the firm's volatility σ, using the firm's stock return volatility after the firm went public. The hidden assumption is that a firm's volatility is not changed before and after the going public process.

5.1 Identification

I estimate the remaining parameters (c_{IPO}, c_{SPAC}, q_{IPO}, q_{SPAC}, ρ, η, π_0, $\mu_b(l)$, γ), using a simulated minimum distance estimator. Similar estimation procedures are widely used (see Bazdresch, Kahn, and Whited (2018)). Given a set of parameters, I solve the model and use the solution to generate a simulated panel of firms with a comparable number of time periods, but with many firms relative to my empirical sample (Michaelides and Ng (2000)). Next, I calculate a set of statistics, which are either moments or functions or moments. I then choose parameter estimates to minimize the distance between the model-generated statistics and their empirical counterparts. To gauge this distance, I use the inverse covariance matrix of the empirical moments. To minimize the econometric objective function, I use a global stochastic optimization routine.

To identify these parameters, I use 10 moment conditions, which I describe in detail below. While each of these moments is related to nearly all model parameters, some moments have strong monotonic relations with certain parameters and are thus particularly useful for identifying those parameters. To ascertain the strength of these relations, I perform a battery of comparative statistics exercises, which I then use to justify my moment choices. The most

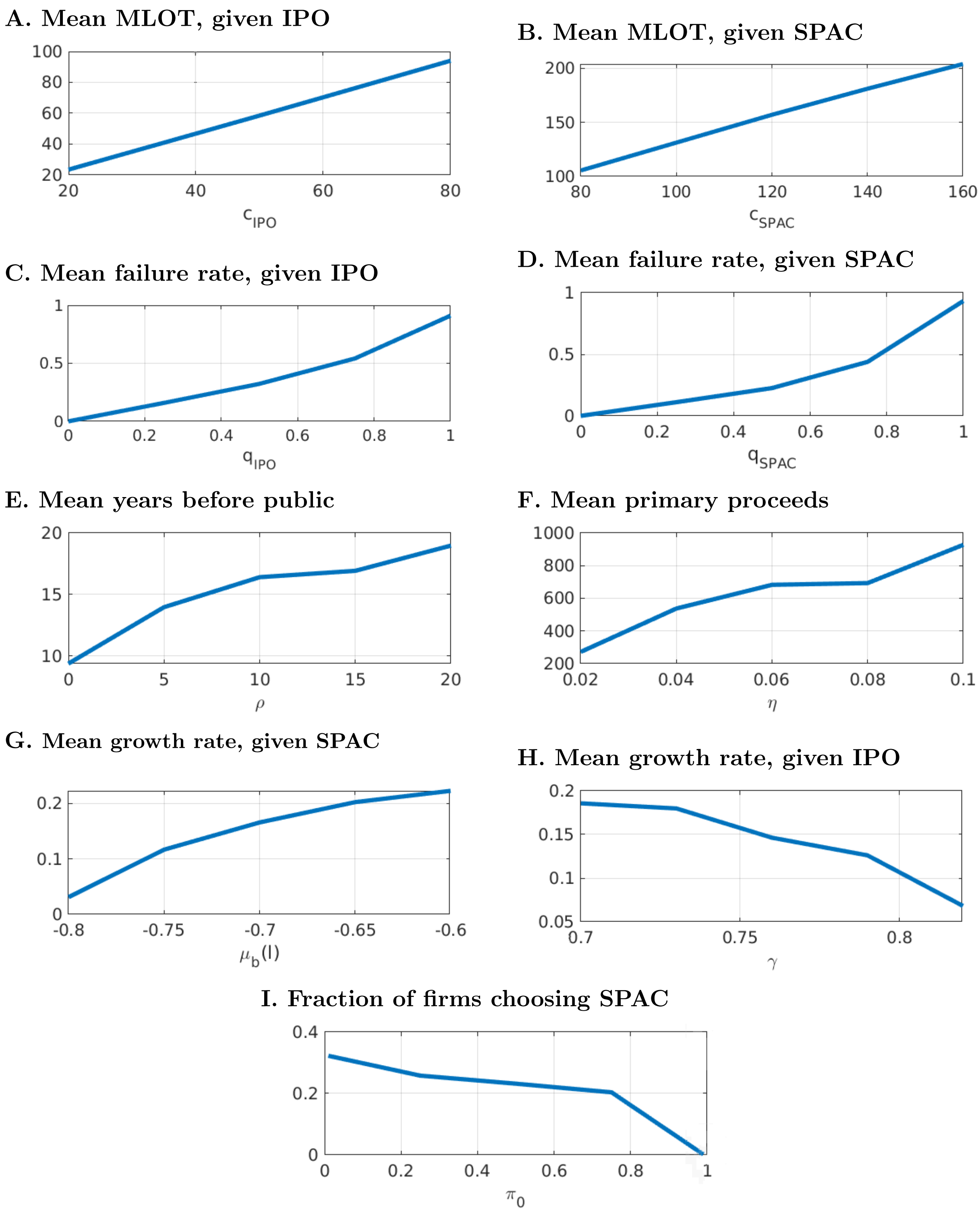

Figure 4: Comparative statistics

Each panel of this figure plots the relation between a moment on the y-axis and a parameter on the x-axis.

relevant of these exercises are in Figure 4.

The cost of generating signals for IPO (c_{IPO}) and SPAC (c_{SPAC}) investors are mainly identified by the average of the Money Left On the Table (MLOT) when firms going the IPO and SPAC process. The higher the cost of the signal-generating process is, the more the firm needs to compensate the public investors by leaving more money on the table.

The precision of signals generated by the IPO (q_{IPO}) and SPAC (q_{SPAC}) investors are mainly identified by the failure rate of going IPO and SPAC. A more precise signal means that more low-type firms are successfully identified by the public signal. Thus, more firms that go public through SPAC will fail the process. In terms of going IPO, it means that more firms that filed an S-1 filing will eventually withdraw from IPO. In terms of going SPAC, it means that more firms that initially signed a contract with a sponsor will eventually terminate the merger process.

The arrival rate of going public opportunity, ρ, is mainly identified by the average time that firms go public. If going public opportunity arrives more frequently, then waiting for the next opportunity is less costly, thus, the average time of going public becomes longer.

The arrival rate of patent expiration, η, measures the frequency at which the founder of the firm can cash out by taking the firm's current earnings. It is mainly identified by the average primary proceeds the firm gets from primary issuing. The intuition is as follows. By taking the firm public, the founder faces a tradeoff: if it goes public, raises additional investment and gets scaled up by a factor of K, at the cost of compensating investors by leaving certain money left on the table today. η here plays the role of discounting the terminal earnings. For a fixed earnings level, the scaling-up effect is larger if the discount rate η is smaller. Thus, when η is larger, the founder understands that the continuation value increase by scaling up is smaller, as a result, the firm is less willing to take the firm public unless the proceeds raised are large enough so the scaling effect is large enough. As a result, when η is larger, the average level of proceeds raised is larger.

The probability that a firm is a high-type firm at the time of founding, π_0, is mainly identified by the fraction of firms going SPAC instead of going IPO. The third point of Lemma 1 shows that firms that prefer SPAC on average have a lower prior belief than firms that prefer going IPO. If the initial belief is lower, then firms have a higher chance of first hitting the boundary where the current belief means that the firm benefits from SPACs most. On the contrary, If π_0 is higher, then firms may have a higher chance of hitting the boundary that the current belief is high enough that going IPO benefits the firm most. To this end, it means that a higher π_0 predicts that more firms will choose IPO over SPAC.

The average growth rate of low-type firms, $\mu_b(l)$, is mainly identified by the average growth rate of firms that remained private. Firms that choose to remain private on average have lower beliefs when they go public, and thus contain more low-type firms compared to firms that chose to go public. For the same reason, The difference in growth rate for high-type and low-type firms, γ, is mainly identified by the average growth rate of firms that chose to go IPO.

In addition to the above moments, I also include the average post-public equity value as an additional moment.

5.2 Estimation Results

The baseline estimation results are in Table 3. Panel A shows the actual data moments, model-simulated moments, and the t-statistics for each paired moment's equality. There's no significant difference between empirical and simulated values for the 10 moments the model addresses, indicating a good model fit.

In Panel B, the parameter estimates reveal the following: The estimated $\mu_b(l) \approx -0.69$ suggests low-type firms have a poor average growth rate. This highlights the importance of scaling decisions for private firms. The estimated γ indicates a 4.5% annual average growth rate for high-type firms, aligning with the growth during economic booms as in Opp (2019).

Table 3
Estimation results

A. Moments

	Data moments	Simulated moments	t-stat
Fraction of firms choosing SPAC	0.292	0.264	-0.40
Failure rate of SPAC	0.166	0.168	0.98
Failure rate of IPO	0.106	0.092	-0.61
Time to going public	17.860	16.991	-0.50
Mean post-public equity value	2,118.400	2,006.800	-0.67
Mean MLOT for SPAC firms	154.780	157.990	1.53
Mean MLOT for IPO firms	56.667	56.456	-1.62
Mean proceeds from primary share selling	240.500	242.31	1.75
Mean growth rate for SPAC	0.147	0.157	1.01
Mean growth rate for IPO	0.188	0.183	-0.84

B. Parameter estimates

$\mu_b(l)$	γ	η	ρ	q_{IPO}	q_{SPAC}	c_{IPO}	c_{SPAC}	π_0
-0.686	0.731	0.048	15.864	0.167	0.293	51.717	121.475	0.083
(0.023)	(0.024)	(0.000)	(0.535)	(0.001)	(0.000)	(0.444)	(1.666)	(0.007)

The estimation is done with a simulated minimum distance estimator, which chooses structural model parameters by matching the moments from a simulated panel of firms to the corresponding moments from the data. Panel A reports the simulated and actual moments and the t-statistics for the differences between the corresponding moments. Panel B reports the estimated structural parameters with standard errors in parentheses.

The results include two Poisson rates. First, η is estimated at 0.048, implying patents expire approximately every 20 years, consistent with the rate in Pástor and Veronesi (2005). The second Poisson rate relates to the frequency of going public opportunities, with the estimated $\rho \approx 16$ suggesting frequent openings in the going public window.

IPO investors detect low-type firms with an accuracy of about 0.16, while SPAC investors have an accuracy of about 0.29. This represents a 75% increase in precision in the SPAC investor's signal compared to the IPO investor's signal.

The model's estimates show that at the point of going public, the prior belief for IPO firms is approximately 0.90, indicating a 90 percent probability of being a high-growth firm. This belief is marginally increased to 0.91 following positive evaluations by the IPO investor, leading to a 4.1% raise in the firm's valuation, or $83 million. In return, the firm leaves the IPO investor $56 million, of which 93% ($52 million) covers the investor's actual cost and the remaining amount is risk markup. In contrast, SPAC investor evaluations play a more substantial role: firms opting for SPAC mergers have a lower prior belief of 0.72 at the time of going public. This belief is elevated to 0.78 after receiving positive evaluations, increasing the firm's equity value by 34.3%, or $539 million. In return, the firm leaves $158 million to investors, of which 76% covers the investor's actual cost and the remaining amount is risk markup.

Finally, the estimated $\pi_0 \approx .08$ shows that firm success rates at inception are below 10%, aligning with typical startup trajectories. Firms often remain private initially, building earnings and investor confidence before entering the market.

After estimating the model, I use it as a laboratory to quantify the actual tradeoffs between choosing an IPO and a SPAC merger. I do so by comparing simulated data between the estimated model and two counterfactual models. In the first counterfactual model, firms that initially choose a SPAC merger are simulated to opt for an IPO instead. I find that these firm's valuation would increase only by 17.3%, or $256 million, but at a cost of $60 million. This suggests that despite the higher costs, the benefits of a more precise evaluation outweight these expenses. In the second counterfactual model, firms that choose IPOs switch to SPAC mergers instead. The simulation results in a 6.7%, or $144 million, increase in their valuation, but incurs a substantial cost of $125 million.

6 Conclusion

This paper develops a dynamic model of the optimal going public choice between IPO and SPAC merger, analyzes the model's predictions, and structurally estimates the model's parameters. In the model, an IPO investor, a SPAC investor and entrepreneurs learn about the mean growth rate of private firms by observing realized earnings. There is no asymmetric information. Entrepreneurs can also learn from investors' costly evaluations. When choosing between SPAC merger and IPO, the entrepreneur faces a tradeoff between the more accurate evaluation provided by the SPAC investor against the higher evaluation cost. The model predicts that firms with high uncertainty–firms' prior beliefs in the middle range–favor SPAC mergers over IPOs as they aim to produce as much information as possible, even at a higher cost. A firm remains private when the prior belief is low, due to a lack of investor interest in evaluating it. A firm prefers IPO when it is confident in being high-growth and faces low uncertainty, as it requires less precise evluation. The model generates simulated moments that match closely to the empirical data moments. According to the model's estimates, for firms opting for SPAC mergers, the SPAC investor's positive evaluation increases the firm's value by 34.3% at a cost of $121 million. In contrast, for firms choosing IPOs, the IPO investor's positive evaluation increases the firm's value by 4.1% at a cost of $52 million.

The main contribution of this paper lies in developing a rational learning framework to explain the choice between an IPO and a SPAC merger. One interesting prediction of the model is the presence of an option value of waiting when firms having high prior beliefs but earnings below the IPO entry threshold. In such cases, a firm's optimal choice depends on its short-term earnings trajectory. If it encoutners several positive earnings shocks in the next few periods, it leans towards an IPO; conversely, several negative earnings shocks would make a SPAC merger more appealing. However, testing this prediction is beyond the scope of this paper due to the lack of reliable proxies for a firm's earnings while it remains private.

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APPENDIX

A.1 Variable Definition

Table A1
Variables Definition

Variable	Definition
Panel A: Model Parameters	
π_0	Initial probability of high-quality firms
B_t	Firm earnings in year t
b_t	Firm-specific log-earnings in year t
o_t	Firm-specific log-odds ratio in year t
$\mu_b(h)$	Drift of b in high firm state ($\theta = h$)
$\mu_b(l)$	Drift of b in low firm state ($\theta = l$)
γ	Difference between $\mu_b(h)$ and $\mu_b(l)$
σ_b	Volatility of b
η	Arrival rate of patent expiration
ρ	Arrival rate of going-public opportunity
K	Scale of post-public earnings
k	Log scale of post-public earnings
c_{IPO}	Information production cost in IPO
c_{SPAC}	Information production cost in SPAC
q_{IPO}	Probability of detecting low type in IPO
q_{SPAC}	Probability of detecting low type in SPAC
r	Interest rate
Panel B: Empirical Variables	
first financing size	Transaction amount of the first financing event. Amounts in millions.
Healthcare	1 if the company operates in the healthcare industry and zero otherwise.
IT	1 if the company operates in the informational technology industry and zero otherwise.
VC reputation	The VC firm's proportion of total investment in the VC industry.
initial employee count	Count of employees that is first available.
industry HHI	The Herfindahl-Hirschman Index.

This table contains defitions of the variables, parameters, and empirical measures used most frequently thoughout the paper. The text contains more detailed defitions

A.2 Institutional Details

In this section, I briefly describe the process of SPAC and discuss the calculation of the total money left on the table (MLOT) by a private firm, and the distinction between the SPAC merger process and the IPO process.

A.1 SPAC and Money Left On the Table (MLOT)

Special Purpose Acquisition Companies (SPACs) have risen as an alternative method for taking private firms public, circumventing the conventional Initial Public Offering (IPO) process. The way a special purpose acquisition company works is that the sponsor of the SPAC raises a pot of money by selling units to public investors at \$10 per unit. Each unit consists of (1) one share and (2) some number of warrants, options to buy another share at some higher price. Generally, the warrant exercise price is \$11.50, a 15% premium to the initial price of the stock, and each unit comes with a fraction of a warrant, often one-quarter or one-third. After the SPAC goes public, the warrants and shares will be traded separately. Say the sponsor offers 20 million units, so the pot of money has \$200 million in it. However, there are not just 20 million SPAC shares outstanding. In fact, the SPAC sponsor will generally give herself 20% ownership of the SPAC – 5 million shares – for free as a reward for her efforts in setting it up and finding a deal. If she fails to find a deal, those shares are canceled: She has to return \$10 per share to the public shareholders, \$200 million total, which uses up all the money in the pot, and there's nothing left for her. In fact, she will lose money, because she paid the SPAC's startup costs, advisory fees, etc, out of her own pocket, which is referred to as the sponsor's "promote". But if she does find a deal, her 5 million shares roll over into shares of the combined company.

After the SPAC's IPO process, the SPAC has a two-year window to identify and negotiate an agreement with a private firm, referred to as the target. During this time, sponsors utilize their industry expertise and connections to find a suitable target. If a suitable target is found, the SPAC and the target company sign an agreement contract, which is similar to the selling contract signed between the underwriter and the private company in the IPO setting. After that, the SPAC is responsible for selling the private company's shares to some large institutional investors, which are referred to as PIPEs, private investments in public equity. The selling process is largely similar to the traditional IPO process. The SPAC sponsor plays the underwriter role, whereas the PIPE investors play the institutional investor role. Despite the differences in terminology, the IPO process and the SPAC merger process share considerable similarities: the sponsor assists the target company in showcasing its business to potential institutional investors via a roadshow. This promotional effort parallels the traditional IPO, with the exception that the firm is permitted to offer forward-looking statements concerning its future plan and growth. Investors subsequently conduct independent assessments of the firm and sign selling agreements to express their commitment to the transaction. The PIPE investors' contract terms are similar to the public investors' contract terms but are also usually \$10 for a share plus some fraction of a warrant. If a deal is successfully negotiated and finalized, the target company is revealed to public investors.

Now let us go back to the simple math. The sponsor finds a private company and they agree to a merger. The SPAC contributes \$200 million, and the PIPE investors contribute another \$200 million. Suppose each SPAC unit contains a quarter warrant, and the public investor and the PIPE investors' contract terms are the same. So let's count the number of shares and warrants held by the sponsor, the public investor, and the PIPE investors. The public investors hold 20 million shares plus 50 million warrants. The sponsor holds 5 million shares. The PIPE investor holds another 20 million shares plus 50 million warrants. In total, there were 45 million shares and 10 million warrants outstanding at the time of the merger announcement held by these three participants. Suppose that after the merger announcement, 50% of the public investors choose to redeem their shares. So they redeem %100 million investment back (suppose the interest rate is zero). Now these three parties contribute \$300 million in total, and they hold 35 million shares and 10 million warrants at the merger's closing date. Now suppose at the SPAC's merger closing date, the post-merger company's share price is \$12, and the warrant's price is \$0.5. We can calculate the money left on the table (MLOT) for each participates. That is, the sponsor makes $\$2 \times 5 = \$10$ million. The public investors make $\$2 \times 10 + \$0.5 \times 5 = \$22.5$ million. The PIPE investors make $\$2 \times 20 + \$0.5 \times 5 = \$42.5$ million. Thus, in total, the firm leaves 75 million dollars on the table.

A.2 Forward-looking Statement and Safe Harbour

One distinct feature of the SPAC merger is that SPAC investors can learn about the company's future business plan. In Appendix A.3, I give an example by comparing investor's presentations of Polestar which merged with SPAC, and Rivian which went IPO. This distinction arises from the protections offered by the Private Securities Litigation Reform Act of 1995 (PSLRA), which do not extend to traditional IPOs. I outline the details of this act below.

The PSLRA, enacted on December 22, 1995, is a critical legislative measure in the United States aimed at reforming the adjudication of securities fraud class-action lawsuits. The PSLRA amended both the Securities Exchange Act of 1934 and the Securities Act of 1933 with the objective of preventing insubstantial, frivolous, or deceptive legal actions that burden the judicial system with considerable expenses. Before the Act's implementation, plaintiffs could sue corporations based on significant and irregular stock price fluctuations in hopes of discovering potential fraud. The PSLRA's safe harbor provision, however, requires plaintiffs to present evidence of reckless or intentional fraudulent statements made by the defendant, proving that their financial losses directly resulted from those statements.

Safe Harbor for Forward-Looking Statements. As part of the PSLRA, U.S. Code § 78u–5 created a "safe harbor" for companies issuing forward-looking statements. To qualify for this safe harbor, a forward-looking statement must be accompanied by cautionary statements identifying critical factors that could cause actual results to deviate materially from those projected in the forward-looking statement. Additionally, these cautionary statements must be tailored specifically to the company and the forward-looking statement in question. This safe harbor provision applies to any forward-looking statement made by a company, including those in SEC filings, earnings calls, press releases, and other public statements. However, the provision does not provide complete immunity for forward-looking statements, as companies can still be held liable for providing misleading information recklessly or intentionally.

Exclusion of IPOs from PSLRA Safe Harbor. Section (b) Exclusions of U.S. Code § 78u–5 detail situations in which the PSLRA safe harbor does not apply unless explicitly provided by the Commission's rule, regulation, or order. One such exclusion refers to forward-looking statements "made in connection with an initial public offering" (IPO). IPOs mark a company's introduction to the public market, serving as the first opportunity for public investors to access the company's business and financial information. As a result, Congress, market participants, analysts, and the SEC staff frequently distinguish IPOs from other capital-raising transactions.

De-SPAC Process Regulations. While SPAC deals share similar purposes to a conventional IPO transaction, they are not excluded from the protection of the PSLRA safe harbor like traditional IPOs. There is growing debate on regulations regarding the de-SPAC process; however, company management is still protected when communicating forward-looking statements during the de-SPAC process. As Chamath Palihapitiya, a former executive at Facebook, stated, "SPAC is a merger of companies; you're all of a sudden allowed to talk about the future; when you do that, you have a better chance of being more fully valued."

The ability to provide additional information and forward-looking statements affords SPACs a unique advantage over conventional IPOs. The SEC website suggests that "Projections are indeed an integral aspect of business combinations. Not only do they help 'sell' the deal, but they also serve as a crucial component for boards and other participants in negotiating and understanding the economics, and ultimately, the fairness of the transaction."

A.3 Summary

Overall, compared to the IPO process, merging with a SPAC has two potential differences. First, SPAC participants may require a potentially higher compensation than IPO participants. Specifically, merging with a SPAC requires the sponsor to initiate a SPAC company in the first place, which is a costly procedure. Second, SPAC investors may have a better chance of fully evaluating the company as they can communicate with the firm about its future production plan and evaluate the firm based on this additional piece of information.

Next, I empirically compare the observed differences for firms that chose to go public through IPO and SPAC.

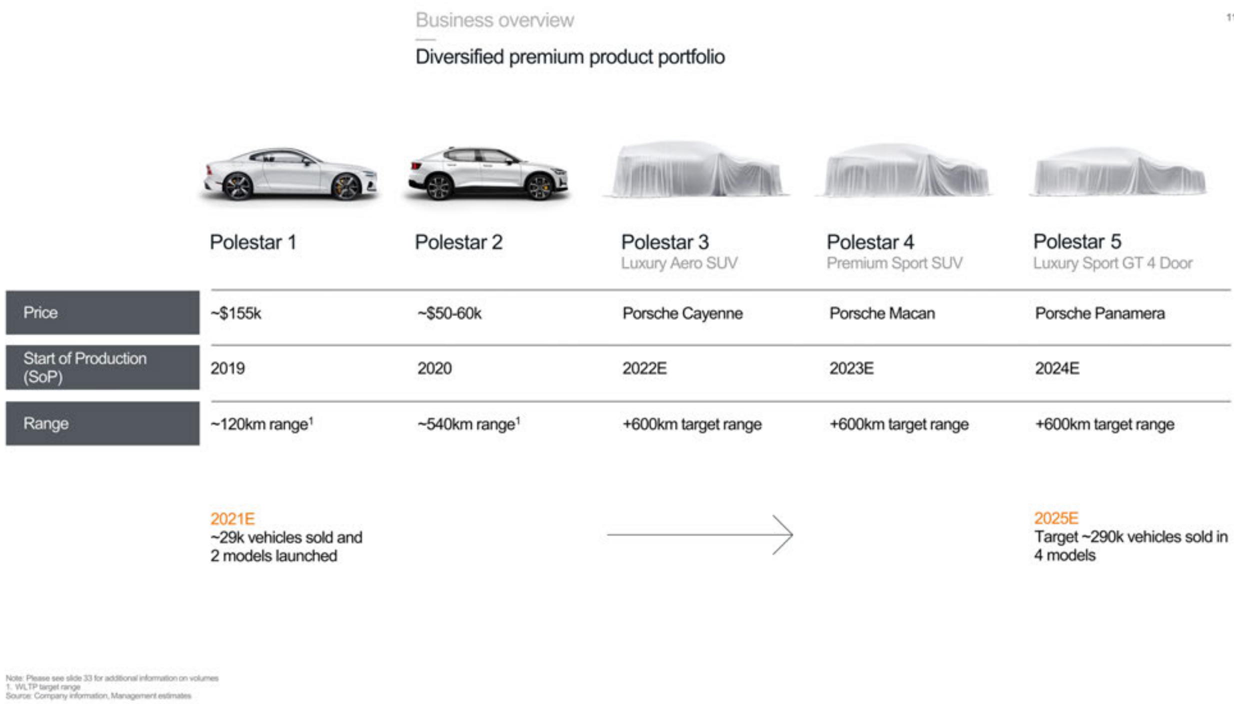
A.3 A Comparison of Investors' Presentations for Rivian and Polestar

In this section, a comparative analysis is performed on two electric vehicle companies that have gone public, emphasizing the investor presentations within their respective prospectuses.

Polestar, a Swedish premium electric performance car brand founded by Volvo Cars and Geely Holding in 2017, pursued going public via a merger with SPAC Gores Guggenheim, Inc. This merger was announced on September 27, 2021, and closed on June 23, 2022, on Nasdaq. Through the SPAC merger, Polestar raised $893.8 million by issuing new shares to public investors, reaching an initial valuation of approximately $26 billion.

On the other hand, Rivian Automotive, Inc., an American electric vehicle manufacturer and automotive technology company established in 2009, chose a traditional IPO route.

Figure A5: Polestar's Prospectus



Polestar presents its future product plan as part of investor's presentation.

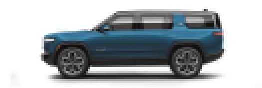
This IPO took place on November 9, 2021, on Nasdaq, raising a significant $11.9 billion and resulting in an initial valuation of approximately $77 billion.

In Polestar's investor presentation, three types of projections are made: future products, market growth, and financial projections. A comparison of these three aspects is provided below:

The first projection concerns the company's plan to introduce new products to the market. At the time of the SPAC merger, Polestar produced two electric performance cars: the hybrid Polestar 1 and the fully electric Polestar 2. Polestar also announced plans to expand its portfolio with three new cars within the next four years, including the Polestar 3 (an electric performance SUV), Polestar 4 (a sport SUV), and Polestar 5 (a sport GT 4-door). Figure A5 illustrates this plan.

In Rivian's prospectus, their production information is based on data up to the end of

Figure A6: Rivian's Prospectus

	R1T	R1S
EPA Rated Range	314 miles (400+ mi. targeted for 2022)	316 miles
Wheelbase	135 in.	121 in.
Length	217 in.	201 in.
Storage	~62 cu. ft.	~105 cu. ft.
Powertrain	800+ horsepower quad motor all-wheel drive	800+ horsepower quad motor all-wheel drive
Acceleration	0-60 mph in ~3 seconds	0-60 mph in ~3 seconds
Towing Capacity	Up to 11,000 lbs.	Up to 7,700 lbs.
Wading Depth	Up to 3 ft.	Up to 3 ft.

Rivian presents its product plan as part of investor's presentation based on products in proudction and signed contracts.

2021, with the IPO taking place in November 2021. Rivian states that it currently has two consumer vehicles available, the R1T and R1S. As of October 31, 2021, the company had approximately 55,400 preorders for R1T and R1S vehicles in the United States and Canada, with customers each placing a cancelable and fully refundable deposit of $1,000. Rivian anticipates fulfilling this preorder backlog of around 55,400 R1 vehicles by the end of 2023. Additionally, Rivian offers a commercial line consisting of EDV 500, EDV 700, and EDV 900 models. Amazon has placed an initial order for 100,000 EDV units, and by the end of 2021, Rivian is expected to produce and deliver approximately 10 EDVs. The company plans to deliver 100,000 EDVs by 2025 and maintain its relationship with Amazon thereafter. Figure A6 illustrates this plan.

In summary, through its SPAC merger, Polestar is able to discuss its future strategies concerning production and financial growth. On the other hand, Rivian, by choosing an IPO, can only provide information about their existing products and sales based on signed contracts.

A.4 Details on the Calculation of Money Left On the Table (MLOT) for SPAC Firms

Table A2
Calculation of SPAC MLOT

	Method
Sponsor	(Share Price − Offer Price) × (Share issued to Sponsor − Share forfeiture)
Public Investor	(Share Price − Offer Price) × Share issued to Public Investor × (1 − Redemption Rate)
	+Warrant Price × Warrant held by Public Investor
PIPE Investor	(Share Price − Offer Price) × Share issued to PIPE Investor
	+Warrant Price × Warrant held by PIPE Investor

This figure summarizes the method of calculating the money left on the table related with going SPAC merger for three participants.

A.5 Proofs

A.1 Martingale Condition of Public Investor's Signal

Here I verify the Martingale condition. At time t, the prior belief about the firm's type is $\Pr(\theta = h|\mathcal{F}_t) = \pi_t$. The Martingale condition is as follows,

$$\Pr(\theta = h|s = H, \mathcal{F}_t)\Pr(s = H) + \Pr(\theta = h|s = L, \mathcal{F}_t)\Pr(s = L) = \Pr(\theta = h|\mathcal{F}_t). \tag{A.1}$$

I can write out each term in the above equation as follows,

$$\begin{aligned}
&\text{Posterior blief with a high signal} \\
=&\Pr(\theta = h|s = H, \mathcal{F}_t) \\
=&\frac{\Pr(\theta = h, s = H|\mathcal{F}_t)}{\Pr(s = H)} \\
=&\frac{\Pr(s = H|\theta = h, \mathcal{F}_t)\Pr(\theta = h|\mathcal{F}_t)}{\Pr(s = H|\theta = h, \mathcal{F}_t)\Pr(\theta = h|\mathcal{F}_t) + \Pr(s = H|\theta = l, \mathcal{F}_t)\Pr(\theta = l|\mathcal{F}_t)} \\
=&\frac{1 \cdot \pi_t}{1 \cdot \pi_t + (1 - q)(1 - \pi_t)},
\end{aligned} \tag{A.2}$$

$$\begin{aligned}
&\text{Posterior blief with a low signal} \\
=&\Pr(\theta = h|s = L, \mathcal{F}_t) \\
=&\frac{\Pr(\theta = h, s = L|\mathcal{F}_t)}{\Pr(s = L)} \\
=&\frac{\Pr(s = L|\theta = h, \mathcal{F}_t)\Pr(\theta = h|\mathcal{F}_t)}{\Pr(s = L|\theta = h, \mathcal{F}_t)\Pr(\theta = h|\mathcal{F}_t) + \Pr(s = L|\theta = l, \mathcal{F}_t)\Pr(\theta = l|\mathcal{F}_t)} \\
=&0,
\end{aligned} \tag{A.3}$$

$$
\begin{aligned}
&\text{Probability of generating a high signal} \\
=&\Pr(s = H) \\
=&\Pr(s = H|\theta = h, \mathcal{F}_t)\Pr(\theta = h|\mathcal{F}_t) + \Pr(s = H|\theta = l, \mathcal{F}_t)\Pr(\theta = l|\mathcal{F}_t) \\
=&1 \cdot \pi_t + (1 - q)(1 - \pi_t),
\end{aligned} \tag{A.4}
$$

$$
\begin{aligned}
&\text{Probability of generating a low signal} \\
=&\Pr(s = L) \\
=&\Pr(s = L|\theta = h, \mathcal{F}_t)\Pr(\theta = h|\mathcal{F}_t) + \Pr(s = L|\theta = l, \mathcal{F}_t)\Pr(\theta = l|\mathcal{F}_t) \\
=&0 + q(1 - \pi_t),
\end{aligned} \tag{A.5}
$$

I can now verify the Martigale condition by calculating the LHS of equation (A.1),

$$
\Pr(\theta = h|s = H, \mathcal{F}_t)\Pr(s = H) + \Pr(\theta = h|s = L, \mathcal{F}_t)\Pr(s = L) \tag{A.6}
$$

$$
=\pi_t \tag{A.7}
$$

$$
=\Pr(\theta = h|\mathcal{F}_t) \tag{A.8}
$$

A.2 Lemma 1

The increase in the posterior belief with a high signal can be represented as the difference in the posterior belief in Equation (A.2) and the prior belief π_t as follows,

$$
\text{Increase in posterior belief with a high signal} \tag{A.9}
$$

$$
\begin{aligned}
=&\Pr(\theta = h|s = H, \mathcal{F}_t) - \Pr(\theta = h|\mathcal{F}_t) \\
=&\frac{1 \cdot \pi_t}{1 \cdot \pi_t + (1 - q)(1 - \pi_t)} - \pi_t
\end{aligned} \tag{A.10}
$$

It can be shown that the above equation is maximized at $\pi_t^* = \sqrt{\frac{1-q}{q}} - \frac{1-q}{q}$. Moreover, it can be show that π_t^* is decreasing in the signal's precision q, with $\pi_t^* = 0.5$ at $q = 0$, and $\pi_t^* = 0$ at $q = 1$.

A.3 Scalablility of HJB of Post-public Equity

Here we show that the HJB equation (13) scales with $B = e^b$. Conjecture that $V_{\text{Post}}(b, o) = e^b \tilde{V}_{\text{Post}}(o)$ and substitues it into equation (13):

$$\begin{aligned} 0 = & - re^b \tilde{V}_{\text{Post}}(o) + \eta(e^b - e^b \tilde{V}_{\text{Post}}(o)) \\ & + \mathbb{E}\left[\lambda_b^+ | o\right] \left[e^{b+\Delta_b} \tilde{V}_{\text{Post}}(o + f^+) - e^b \tilde{V}_{\text{Post}}(o)\right] \\ & + \mathbb{E}\left[\lambda_b^- | o\right] \left[e^{b-\Delta_b} \tilde{V}_{\text{Post}}(o - f^-) - e^b \tilde{V}_{\text{Post}}(o)\right]. \end{aligned} \tag{A.11}$$

Factor out e^b:

$$\begin{aligned} 0 = & e^b \Big(-r\tilde{V}_{\text{Post}}(o) + \eta(1 - \tilde{V}_{\text{Post}}(o)) \\ & + \mathbb{E}\left[\lambda_b^+ | o\right] \left[e^{\Delta_b} \tilde{V}_{\text{Post}}(o + f^+) - \tilde{V}_{\text{Post}}(o)\right] \\ & + \mathbb{E}\left[\lambda_b^- | o\right] \left[e^{-\Delta_b} \tilde{V}_{\text{Post}}(o - f^-) - \tilde{V}_{\text{Post}}(o)\right] \Big). \end{aligned} \tag{A.12}$$

Since e^b is never zero, we can divide both sides by e^b:

$$\begin{aligned} 0 = & - r\tilde{V}_{\text{Post}}(o) + \eta(1 - \tilde{V}_{\text{Post}}(o)) \\ & + \mathbb{E}\left[\lambda_b^+ | o\right] \left[e^{\Delta_b} \tilde{V}_{\text{Post}}(o + f^+) - \tilde{V}_{\text{Post}}(o)\right] \\ & + \mathbb{E}\left[\lambda_b^- | o\right] \left[e^{-\Delta_b} \tilde{V}_{\text{Post}}(o - f^-) - \tilde{V}_{\text{Post}}(o)\right]. \end{aligned} \tag{A.13}$$

The expression inside the parentheses is now a function of o only. We can multiply both sides by e^{kb} and get

$$\begin{aligned}0 =& e^{kb}\left(-r\tilde{V}_{\text{Post}}(o)+\eta(1-\tilde{V}_{\text{Post}}(o))\right.\\&+\mathbb{E}\left[\lambda_b^+|o\right]\left[e^{\Delta_b}\tilde{V}(o+f^+)-\tilde{V}_{\text{Post}}(o)\right]\\&\left.+\mathbb{E}\left[\lambda_b^-|o\right]\left[e^{-\Delta_b}\tilde{V}_{\text{Post}}(o-f^-)-\tilde{V}_{\text{Post}}(o)\right]\right).\end{aligned}\tag{A.14}$$

This indicates that $V_{\text{Post}}(kb,o)$ also solves the HJB equation.

A.4 Lemma 2

I first derive the Bayesian updating formula associated with the earnings shocks.

Restriction (3) implies the following equations:

$$\lambda_b = \lambda_b^+(h)+\lambda_b^-(h),\tag{A.15}$$

$$\lambda_b = \lambda_b^+(l)+\lambda_b^-(l),\tag{A.16}$$

Combining the above equations with the formulas for $\mu_b(\theta)$ and σ_b provided in equations (4) and (5), we can express Poisson intensities as follows

$$\begin{aligned}\lambda_b^+(h) =&\frac{\sigma_b^2}{2\Delta_b^2}\left(1+\Delta_b\frac{\mu_b(h)}{\sigma_b^2}\right)\\\lambda_b^+(l) =&\frac{\sigma_b^2}{2\Delta_b^2}\left(1+\Delta_b\frac{\mu_b(l)}{\sigma_b^2}\right)\\\lambda_b^-(h) =&\frac{\sigma_b^2}{2\Delta_b^2}\left(1-\Delta_b\frac{\mu_b(h)}{\sigma_b^2}\right)\\\lambda_b^-(l) =&\frac{\sigma_b^2}{2\Delta_b^2}\left(1-\Delta_b\frac{\mu_b(l)}{\sigma_b^2}\right)\end{aligned}$$

By definition, Bayes factors associated with positive and negative innovations are given by

$$e^{f^+} \equiv \frac{\lambda_b^+(h)}{\lambda_b^+(l)} = \frac{1+\Delta_b \frac{\mu_b(h)}{\sigma_b^2}}{1+\Delta_b \frac{\mu_b(l)}{\sigma_b^2}}, \tag{A.17}$$

$$e^{f^-} \equiv \frac{\lambda_b^-(h)}{\lambda_b^-(l)} = \frac{1-\Delta_b \frac{\mu_b(h)}{\sigma_b^2}}{1-\Delta_b \frac{\mu_b(l)}{\sigma_b^2}}. \tag{A.18}$$

Next, I examine the updating of the log-odds ratio with the incorporation of a signal generated by the public investor. The objective is to compute the log-odds ratio based on posterior probabilities. I start with calculating the posterior log-odds ratio when the generated signal is good:

$$\begin{aligned}
&\log\left[\frac{\Pr(\theta=h|s=H,\mathcal{F}_t)}{\Pr(\theta=l|s=H,\mathcal{F}_t)}\right] \\
=&\log\left[\frac{\frac{1\cdot\pi_t}{1\cdot\pi_t+(1-q)(1-\pi_t)}}{1-\frac{1\cdot\pi_t}{1\cdot\pi_t+(1-q)(1-\pi_t)}}\right] \\
=&\log\left[\frac{\pi_t}{(1-q)(1-\pi_t)}\right] \\
=&\log\left[\frac{\pi_t}{1-\pi_t}\right]+\log\left[\frac{1}{1-q}\right] \\
=&o_t+\log\left[\frac{1}{1-q}\right] \\
\equiv&o_t+f \qquad ,
\end{aligned} \tag{A.19}$$

where

$$\begin{aligned}
o_t &= \log\left[\frac{\Pr(\theta=h|\mathcal{F}_t)}{\Pr(\theta=l|\mathcal{F}_t)}\right] = \log\left[\frac{\pi_t}{1-\pi_t}\right], \\
f &\equiv \log\left[\frac{1}{1-q}\right].
\end{aligned}$$

Subsequently, I compute the posterior log-odds ratio when the generated signal is bad:

$$\begin{aligned}
&\log\left[\frac{\Pr(\theta=h|s=L,\mathcal{F}_t)}{\Pr(\theta=l|s=L,\mathcal{F}_t)}\right]\\
=&\log\left[\frac{0}{1-0}\right]\\
=&-\infty. \qquad \text{(A.20)}
\end{aligned}$$

The above results reveal that when the public investor receives a good signal, the log-odds ratio increases by $f=\log\left[\frac{1}{1-q}\right]$. Conversely, when the public investor receives a bad signal, the log-odds ratio declines to $-\infty$.

A.6 Solution Alogorithm

A.1 Example of Solving HJB in Matrix Form

In this section, I give a simple example explaining how to solve a HJB. Consider the simpliest case with one state variable. The current state is s, and the next state can take two possible values, $s+f^+$ and $s-f^-$, with probabilities λ^+ and λ^-. we can write out the HJB function for each of the state variable.

$$rW(s)=\psi(s)+\lambda^+[W(s+f^+)-W(s)]+\lambda^-[W(s-f^-)-W(s)],$$

We assume the simpliest case that $f^+=f^-=\Delta_s$, and s takes values of $\{0,\Delta_s,2\Delta_s\}$. We can write out of a linear system of equations as follows,

$$\begin{aligned}
rW(0)&=\psi(0)+\lambda^+[W(\Delta_s)-W(0)]+\lambda^-[W(0)-W(0)],\\
rW(\Delta_s)&=\psi(\Delta_s)+\lambda^+[W(2\Delta_s)-W(\Delta_s)]+\lambda^-[W(0)-W(\Delta_s)],\\
rW(2\Delta_s)&=\psi(2\Delta_s)+\lambda^+[W(2\Delta_s)-W(2\Delta_s)]+\lambda^-[W(\Delta_s)-W(2\Delta_s)].
\end{aligned}$$

Now we can change notation and write the above system of equations in matrix form as

$$r\begin{bmatrix} W(0) \\ W(\Delta_s) \\ W(2\Delta_s) \end{bmatrix} = \begin{bmatrix} \psi(0) \\ \psi(\Delta_s) \\ \psi(2\Delta_s) \end{bmatrix} + \begin{bmatrix} \lambda_{0,0} & \lambda_{0,\Delta_s} & \lambda_{0,2\Delta_s} \\ \lambda_{\Delta_s,0} & \lambda_{\Delta_s,\Delta_s} & \lambda_{\Delta_s,2\Delta_s} \\ \lambda_{2\Delta_s,0} & \lambda_{2\Delta_s,\Delta_s} & \lambda_{2\Delta_s,2\Delta_s} \end{bmatrix} \begin{bmatrix} W(0) \\ W(\Delta_s) \\ W(2\Delta_s) \end{bmatrix}.$$

Filling in the transition rates, we have,

$$r\begin{bmatrix} W(0) \\ W(\Delta_s) \\ W(2\Delta_s) \end{bmatrix} = \begin{bmatrix} \psi(0) \\ \psi(\Delta_s) \\ \psi(2\Delta_s) \end{bmatrix} + \begin{bmatrix} -\lambda^+ & \lambda^+ & 0 \\ \lambda^- & -\lambda^+ - \lambda^- & \lambda^+ \\ 0 & \lambda^- & -\lambda^- \end{bmatrix} \begin{bmatrix} W(0) \\ W(\Delta_s) \\ W(2\Delta_s) \end{bmatrix}.$$

Now we can group the value functions together and put it on the right, and put the cash flows on the left, and rewrite as follows,

$$-\begin{bmatrix} \psi(0) \\ \psi(\Delta_s) \\ \psi(2\Delta_s) \end{bmatrix} = \begin{bmatrix} -r-\lambda^+ & \lambda^+ & 0 \\ \lambda^- & -r-\lambda^+ - \lambda^- & \lambda^+ \\ 0 & \lambda^- & -r-\lambda^- \end{bmatrix} \begin{bmatrix} W(0) \\ W(\Delta_s) \\ W(2\Delta_s) \end{bmatrix}.$$

Finally, using matrix inverting, we can solve for the value functions as

$$\begin{bmatrix} W(0) \\ W(\Delta_s) \\ W(2\Delta_s) \end{bmatrix} = \begin{bmatrix} -r-\lambda^+ & \lambda^+ & 0 \\ \lambda^- & -r-\lambda^+ - \lambda^- & \lambda^+ \\ 0 & \lambda^- & -r-\lambda^- \end{bmatrix}^{-1} \left(-\begin{bmatrix} \psi(0) \\ \psi(\Delta_s) \\ \psi(2\Delta_s) \end{bmatrix} \right).$$

Chaning notation, we have,

$$\mathbf{W} = \mathbf{Q}^{-1}\left(-\boldsymbol{\psi}\right)$$

Solution algorithm:

1. Define $\mathbf{Q}$ as an empty sparse matrix of size $n \times n$.

2. Populate the matrix $\mathbf{Q}$ with transitions and discount rates.

3. Compute the vector $\boldsymbol{\psi}$ given the state vector.

4. Compute $\mathbf{W}$ using matrix inversion.

A.2 Solving Post-Public Value Function

The system of HJB equations can be represented as follows:

$$0 = -\psi_1(b, o) - (\eta + r)W(b, o) + \Lambda\mathbf{W},$$

where Λ is an $n \times n$ transition matrix, and $\mathbf{W}$ is an $n \times 1$ vector of value function values, and where the local expected value at the expiration date is $\psi_1(b, o) = \eta e^b$.

We can further rewrite the system of HJB equations as follows:

$$-\boldsymbol{\psi}_1 = Q_1 \cdot \mathbf{W},$$

where Q_1 is a matrix that is equal to the transition matrix Λ except that it includes discounting by r and η on the disagonal. The value functions $\mathbf{W}$ solve the following linear system for any given set of state vectors:

$$\mathbf{W} = Q_1^{-1} \cdot (-\boldsymbol{\psi}_1) \tag{A.21}$$

<u>Solution algorithm:</u>

1. Define Q_1 as an empty sparse matrix of size $n \times n$.

2. Populate the matrix Q_1 with exogenous transitions and discount rates.

(a) For positive earnings shocks, map (b, o) to $(b + \Delta_b, o + f^+)$ using transition rates $\mathbb{E}\left[\lambda_b^+|o\right] = \pi(o)\lambda_b^+(h) + (1 - \pi(o))\lambda_b^+(l)$, where $f^+ = \alpha^+\Delta_o$, and α^+ is a natural number by Assumption 1.

(b) For negative earnings shocks, map (b, o) to $(b - \Delta_b, o - f^-)$ using transition rates $\mathbb{E}\left[\lambda_b^-|o\right] = \pi(o)\lambda_b^-(h) + (1 - \pi(o))\lambda_b^-(l)$, where $f^- = \alpha^-\Delta_o$, and α^- is a natural number by Assumption 1.

(c) Add discounting rates r and η to the diagonal terms.

3. Compute the vector $\boldsymbol{\psi}_1$ given the state vector.

4. Compute $\mathbf{W}$ based on equation (A.21).

A.3 Solving Going-Public Value Function

The system of HJB equations can be represented as follows:

$$0 = -\psi_2(b, o) - (\rho + \eta + r)V(b, o) + \Lambda\mathbf{V},$$

where Λ is an $n \times n$ transition matrix, and $\mathbf{V}$ is an $n \times 1$ vector of value function values, and where the local expected value at the expiration date is $\psi_2(b, o) = \eta e^b + \rho V_{max}(b, o)$, where V_{max} is as follows,

$$V_{max}(b, o) = \max\{V_{Public}(b, o), V(b, o)\},$$

where

$$V_{Public}(b, o) = [\pi(o) + (1 - q)\pi(o)]\left[p(b, o)(1 - \alpha) - \left(e^{k\Delta_b} - 1\right)e^b\right] + [q(1 - \pi(o))]\,V(b, -\infty),$$

and where

$$p(b, o) = W(o + f, b + k\Delta_b) - \frac{c}{\pi(o) + (1 - q)(1 - \pi(o))}.$$

We can further rewrite the system of HJB equations as follows:

$$\boldsymbol{\psi}_2 = Q_2 \cdot \mathbf{V},$$

where Q_2 is a matrix that is equal to the transition matrix Λ except that it includes discounting by r, η, and ρ on the disagonal. The value functions $\mathbf{V}$ solve the following linear system for any given set of state vectors:

$$\mathbf{V} = Q_2^{-1} \cdot (-\boldsymbol{\psi}_2) \tag{A.22}$$

<u>Solution algorithm:</u>

1. Define Q_2 as an empty sparse matrix of size $n \times n$.
2. Populate the matrix Q_2 with exogenous transitions and discount rates.
   (a) For positive earnings shocks, map (b, o) to $(b + \Delta_b, o + f^+)$ using transition rates $\mathbb{E}\left[\lambda_b^+ | o\right] = \pi(o)\lambda_b^+(h) + (1 - \pi(o))\lambda_b^+(l)$, where $f^+ = \alpha^+\Delta_o$, and α^+ is a natural number by Assumption 1.
   (b) For negative earnings shocks, map (b, o) to $(b - \Delta_b, o - f^-)$ using transition rates $\mathbb{E}\left[\lambda_b^- | o\right] = \pi(o)\lambda_b^-(h) + (1 - \pi(o))\lambda_b^-(l)$, where $f^- = \alpha^-\Delta_o$, and α^- is a natural number by Assumption 1.
   (c) Add discounting rates r, η, and ρ to the diagonal terms.
3. Compute the vector $\boldsymbol{\psi}_2$ given the state vector.
4. Compute $\mathbf{V}$ based on equation (A.22).