Tick Size Constraints, Two-Sided Markets, and Competition between Stock Exchanges

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Abstract

U.S. exchange operators compete for order flow by setting "make" fees for limit orders ("makers") and "take" fees for market orders ("takers"). When traders can quote continuous prices, the manner in which operators divide the total fee between makers and takers is irrelevant because traders can choose prices that perfectly counteract any division of the fee. The one cent minimum tick size imposed by SEC 612 to traders prevents perfect neutralization and also destroys mutually agreeable trades at price levels that range within a tick. These frictions 1) create both scope and incentive for an operator to establish multiple platforms that differ in fee structure in order to engage in second-degree price discrimination, and 2) lead to mixed-strategy equilibria with positive profits for competing operators, rather than to zero-fee, zero-profit Bertrand equilibrium. We show that price discrimination via platforms with differing fees can Pareto-improve social welfare in the presence of tick-size constraints. Our model predicts that markets become more fragmented under a larger tick size. We find empirical evidence consistent with this prediction using splits/reverse splits of ETFs as exogenous shocks to the relative tick size, with paired ETFs that track the same index but do not split/reverse split as controls.

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Technological advances have changed the nature of stock exchanges. Trades used to occur through the intermediation of dealers or specialists in “discrete time.” With the advent of electronic trading, stock exchanges in the U.S. have become electronic limit-order books, such that trades happen through direct interaction between buyers and sellers, and at a much higher speed, than was previously possible. Along with this change we have witnessed the proliferation of stock exchanges, which fragments trading volume. Figure 1 displays three major holding companies of stock exchanges (which we refer to as “operators”), each of which operates multiple exchanges (which we refer to as “platforms”). [1] These competing platforms offer nearly homogeneous trading services. First, the same stock can be traded on any of these ten platforms because U.S. regulation allows stocks to be traded outside the listing venue. Second, these platforms are organized mainly as electronic limit-order books. A trader can act as a liquidity maker by posting a limit order with a specified price and quantity, and a trade happens once another trader (taker) accepts the terms of a previously posted limit order through a market order. Third, these platforms adopt the make-take fee pricing model, for which the liquidity maker pays a “make” fee and the liquidity taker pays a “take” fee on each executed share (we treat rebates as negative fees). The sum of the make fee and the take fee, the so-called “total” fee, is a major source of profit for these platforms.[2]

We argue that a discrete tick size is one driving force behind the make-take fee pricing model, and the fragmentation of trading across operators and among platforms belonging to the same operator. When traders can quote continuous prices, the tax-neutrality principle predicts that platforms should compete only on the total fee, not on how the total fee is broken down into the make fee and take fee, as traders are able to neutralize the make and take fee allocations by adjusting their quotes. The liquidity maker in the stock exchange, however, cannot propose orders in increments smaller than one cent for any stock priced above $1.00 per share due to Security and

[1] During the sample period of Figure 1, the only active exchange that does not belong to these three groups is the Chicago Stock Exchange, which accounts for less than 1% of the market share in trading volume.
[2] For example, in its filing for an IPO, the BATS stock exchange reports that about 70% percent of its revenues come from the total fee. BATS S1 registration statement (page F4). O’Donoghue estimate that 34.7% of NASDAQ’s net income is from the fees.

Exchanges Commission (SEC) rule 612 of regulation National Market Systems (NMS). The make-take fees set by platforms are, however, not subject to the tick size constraints. Consequently, platforms can use make-take fees to effectively propose sub-penny transaction prices that cannot be neutralized by liquidity makers. Therefore, the discrete tick size changes the nature of price competition between platforms from one-sided (total fee) competition to two-sided (make fee and take fee) competition, which in turn leads to two economic forces that fragment the market. First, an operator has incentive to establish multiple platforms that differ in fee structure in order to engage in second-degree price discrimination. Second, competition on two sides generates positive profits for identical platforms that competes on price, which encourages new entry.

The following example illustrates the intuition behind our theoretical model. Consider a game between exchange operator(s), a continuum of buyers with valuations uniformly distributed on [0.5, 1] and a continuum of sellers with valuations uniformly distributed on [0, 0.5]. At Date 0, the profit-maximizing operator(s) move(s). In the monopoly case, the operator makes two decisions: how many platforms to establish and how to structure the fees on each platform. In the duopoly case, two operators simultaneously determine their fee structures on their platforms. The trading stage of the game proceeds in the same way in the monopoly and duopoly cases. A liquidity maker arrives at Date 1. Without loss of generality, we consider the case in which the liquidity maker arrives as a buyer. The liquidity maker chooses a platform and a price at which she posts her limit order of 1 share. By doing so, the liquidity maker chooses the cum fee buy price of her limit order, which is the limit order price plus the make fee charged by the platform. The liquidity maker has the option of posting no limit orders and leaves all platforms empty. At Date 2, a liquidity taker arrives. She decides whether to accept the limit order on the platform selected by the liquidity maker by comparing her own valuation with the cum fee sell price of her market order, which is the buy limit order price minus the take fee charged by the platform. The platform collects the make and take fees upon matching the two orders.

When the liquidity maker can quote a continuous price, our model makes three prediction consistent canonical economic principles. 1) Consistent with tax-neutrality principle, we find platforms compete only on the total fee but not the breakdown of the make fee and take fee. The one-dimensional competition for the total fee together with homogeneous trading services lead then to two follow-on predictions. 2) No price discrimination: operators have no incentive to open multiple platforms, because all traders would choose the platform with the lowest total fee. 3) Bertrand outcome: competition between operators ends in pure-strategy equilibrium with zero total fees and zero profits. These two predictions then imply consolidation of trading platforms if setting up a platform involves fixed costs.

Next, consider tick size is 1 and the liquidity maker can quote *only integers*. Then the liquidity maker cannot quote a price within the tick, but a platform can create differentiated sets of sub-tick cum fee limit-order and market-order prices by changing the fee structure. This non-neutrality is first discovered by Foucault, Kadan and Kandel (2013) under one operator with one platform. We advance their intuition by showing that non-neutrality creates vertical product differentiation for otherwise identical platforms. A liquidity taker is more likely to accept the liquidity maker's limit order in a platform with a better cum fee price. Therefore, platforms with heterogeneous take fees are vertically differentiated: a platform with a better cum fee price for the liquidity taker is of higher quality for the liquidity maker, because such platform offers a higher probability for liquidity makers to realize their gains from trade. The operators' choice of make and take fees at stage 0, from the point of the view of the liquidity maker, is equivalent to a simultaneous choice of price of execution service (the make fee) and quality of execution service (the take fee).

This vertical product differentiation then facilitates second-degree price discrimination by a monopoly operator. All liquidity makers prefer a platform with higher quality, but they differ in their willingness to pay for the quality. This allows the operator to open multiple platforms with differentiated prices and execution probabilities. The liquidity makers then self-select based on their gains from trade. Liquidity makers with high gains from trade select the platform with the higher cum fee buy price and the higher cum fee sell price (or execution probability). Liquidity makers with low gains from trade select the platform with the lower cum fee buy price and the lower cum fee sell price (or execution probability). More interestingly, we show that such second-degree price discrimination increases not only the operator's profit, but also the welfare of liquidity makers and liquidity takers, because adding platforms creates more effective transaction prices for end users.

The simultaneous choice of price and quality by duopoly operators under tick size constraints destroy not only Bertrand equilibrium but also any pure-strategy equilibrium. There exists no pure-strategy equilibrium with positive total fees, because competing operators have incentives to undercut each other toward zero total fees. The additional insight from the discrete tick size, however, is that Bertrand equilibrium with zero total fees cannot be sustained either. Given one operator charging a zero total fee, there are two types of profitable deviations for the other platform which increase the total fee. One type of strategy charges a liquidity maker ε more while charging a liquidity taker $\mu \cdot \varepsilon$ less (where $0 < \mu < 1$). Such a deviation reduces a liquidity maker's profit conditional on execution, but meanwhile increases the execution probability, which attracts liquidity makers with higher trading surpluses. The other type of strategy attracts liquidity makers with lower trading surpluses, by charging a liquidity maker $\mu \cdot \varepsilon$ less and a liquidity taker ε more. Importantly, we show that under symmetrical mixed-strategy equilibria both platforms earn strictly positive profits, which explains the new entry into the fee game. The fact that only mixed strategy equilibria exist also rationalizes the diversity of fee structures and the frequent fee adjustments that have been observed empirically (O'Donoghue (2014) and Cardella, Hao, and Kalcheva (2012)).

We contribute to the literature on make-take fees by proposing the first platform competition model with a discrete tick size. Colliard and Foucault (2012) assume a zero tick size, and they show that platforms compete only on total fees and that the competition leads to a Bertrand outcome. Yet the empirical result by Cardella, Hao, and Kalcheva (2012) demonstrate that total fees do not converge on a stable value, let alone on zero, as in the Bertrand outcome.[3] The mixed strategy equilibrium rationalizes the diverse fee structure and their frequent changes documented in their paper. Skjeltorp, Sojli and Tham (2011) find empirical evidence that

[3] Figure 3 on p. 42 of Cardella, Hao, and Kalcheva (2012). The paper is available at http://www.fma.org/Atlanta/Papers/50.fees2012.01.01paper.pdf

make/take fees create price discrimination. However, Foucault (2012) raises the puzzle that "it is not clear however how the differentiation of make/take fees suffices to screen different types of investors." We addresses this puzzle by proposing a new form of second-degree price discrimination: when end users cannot neutralize the breakdown of the fee, and the operators can screen liquidity makers based on the terms of trade offered to the liquidity takers.

Our results also helps in evaluating a recent policy initiative to completely ban these fees.[4] One argument in favor of banning the fees cites their complexity and frequent fluctuations, but this complexity can be justified by the mixed-strategy equilibria documented in this paper.[5] The other criticism on the fee is based on fairness, because the fee leads to wealth transfer from the side paying the fee to the side being subsidized.[6] However, we show that liquidity providers prefer being charged instead of being subsidized when the tick size is large, and vice versa when the tick size is small. This counterintuitive result is generated by two "costs" of the subsidy. First, a subsidy given to a liquidity maker is from the take fee imposed on a liquidity taker. A high take fee can reduce the probability that a liquidity taker accepts the limit order and that a liquidity maker realizes gains from a trade. Second, a subsidy given to a liquidity maker can force her to quote a more aggressive price, which can lead to a less favorable cum fee price and reduce the gains from trade. With a fixed fee level, the cost of the subsidy is higher when the tick size is larger. These two economic mechanisms provide a plausible interpretation for the existence of taker/maker markets (platforms subsidizing takers and charging makers).

Our paper also contributes to the literature on market fragmentation. The literature generally predicts consolidation of trading due to network externality or economies of scale.[7] Yet O'Hara and Ye (2011) demonstrates significant fragmentation of trading volume. We reveals two economic forces that fragments the market: second degree price discrimination and positive profit

[4] "Make-take fees in spotlight on Capitol Hill." http://marketsmedia.com/make-take-fees-spotlight/.
[5] See the argument by Tom Farley, president of Intercontinental Exchange's NYSE Group on market complexity, in "Make-take fees in spotlight on Capitol Hill."
[6] See the discussion in Malinova and Park (2015).
[7] See Pagano (1989), Admati and Pfleiderer (1988). Biais, Glosten and Spatt (2005), Stigler (1961, 1964), Doede (1967), Demsetz (1968) and Chowdhry and Nanda (1991).

led by two-sided competition. To the best of our knowledge, the price discrimination channel has not been theoretically examined. For the second channel, extant literature on exchange competition is based mostly on exogenous exchanges or exchanges offering differentiated products.[8] One exception is Foucault and Parlour (2004), which shows that competing platforms can co-exist by offering differentiated listing fees and trading costs. We show that otherwise-identical trading platforms can co-exist even if they compete on trading but not on listing.

The prediction that that the tick size constraints encourage fragmentation in stock trading is tested by the following identification strategy. We use ETF splits/reverse splits as exogenous shocks to the relative tick size (one divided by the price), with ETFs that split/reverse split as the treatment group and with ETFs that track the same index but experience no splits/reverse splits as the control group. We find that splits fragment trading volume and reverse splits consolidate trading volume.

Lastly, this paper contributes to the burgeoning literature on two-sided markets. Two-sided markets are markets in which "the volume of transactions between end-users depends on the structure and not only on the overall level of the fees charged by the platform" (Rochet and Tirole (R&T hereafter), (2006), p. 646). A fundamental challenge to the two-sided markets literature is to demonstrate that two-sidedness can generate qualitatively different predictions from those with identical setups except for one-sidedness. Our paper nests a two-sided model in a one-sided model, and finds that operators of a two-sided market can engage in a more complex pricing strategy compared with operators of a one-sided market. In our model, two-sidedness creates product differentiation between the two intrinsically homogeneous platforms, which in turn creates second-degree price discrimination, destroys any pure-strategy equilibrium, and leads to market fragmentation. These stark contrasts in price competition and the resulting market structure confirm the value of investigating two-sided markets independently.

[8] Two exchanges can co-exist in Colliard and Foucault (2012) only when there is no cost to establish an exchange. For models based on exogenous exchanges, see Glosten (1994), Parlour and Seppi (2003), Hendershott and Mendelson (2000), and Foucault and Menkveld (2008). For the product differentiation model, see Pagnotta and Philippon (2013), Santos and Scheinkman (2001), and Rust and Hall (2003), among others.

Also, the literature on two-sided markets is overwhelmingly based on network externality with multiple sides (Rysman, (2009)).[9] Our paper identifies another competing force to consider with regard to two-sided markets: product differentiation due to non-neutrality. In our model, operators simultaneously choose both the price and quality of the platform for the liquidity maker by setting the make and take fees. Such a simultaneous choice of price and quality leads to mixed-strategy equilibrium, contrary to models that are based on a sequential move of choosing the quality first and then choosing the price (Shaked and Sutton, (1982)).

The rest of the paper is organized as follows. Section I sets up the model. Section II examines the non-neutrality of the fee under a discrete tick size. Section III examines the product differentiation due to the non-neutrality of the fees. Section IV considers second-degree price discrimination. Section V considers competing platforms with duopoly operators. Section VI presents the empirical tests of our theoretical predictions. Section VII concludes the paper and discusses the policy implications. The appendix contains mathematical proofs of the lemmas and propositions.

I. Model

A. Model Setup

Our model includes three types of risk-neutral players. Exchange operator(s), a continuum of liquidity makers with valuations of a stock v_b uniformly distributed on $[d/2,\ d]$, and a continuum of liquidity takers with valuations of the stock v_s uniformly distributed on $[0,\ d/2]$. v_b and v_s , respectively, are the liquidity maker's and the liquidity taker's private information. Because a liquidity maker has a higher valuation than a liquidity taker, a liquidity maker intends to buy from a liquidity taker. The results when the liquidity maker intends to sell to the liquidity taker are the same because of symmetric valuation and uniform distribution. (not reported for brevity). We consider both the case of one monopoly operator and the case of duopoly operators. The game has three stages. At Date 0, the operator(s) move. The operator in the monopoly case

[9]See Rochet and Tirole (2003, 2006), Armstrong (2006), Armstrong and Wright (2007), Evans and Schmalensee (2007, 2013), and Chao and Derdenger (2013).

makes two decisions: determining the optimal number of platforms to establish and setting the fee structure $F^i = (f_m^i, f_t^i)$ on each platform, where f_m^i denotes the make fee for a liquidity maker and f_t^i denotes the take fee for a liquidity taker. A negative fee in the model implies a subsidy. The operators in the duopoly case simultaneously choose their fee structures at Date 0. For simplicity, we assume that each operator can establish only one platform. Fees are charged only upon trade execution. The trading stage of the game under both a monopoly operator and duopoly operators proceeds in the same way. At Date 1, nature draws a liquidity maker with valuation v_b. The liquidity maker makes two decisions after observing the fee structures: choosing the exchange in which she submits a limit order for one share and determining the price P of the limit order. The liquidity maker is allowed to submit no limit order at all. At Date 2, a liquidity taker arrives. The liquidity taker observes the make and take fees as well as the price proposed by the liquidity maker, and then she decides whether to trade. If she decides to trade, she must join the platform that the liquidity maker chooses at Date 1 and trade at the proposed trading price P.[10] Once a trade happens, the platform profits from the total fee (the sum of the make fee f_m^i and the take fee f_t^i).

Since the purpose of this paper is to examine the impacts of tick size constraints on market outcomes in this model, we consider two extreme tick sizes: a continuous tick size of 0 and a discrete tick size of d. Other tick sizes can be considered as intermediary cases between these two.[11] A liquidity maker can propose any price under a continuous tick size, but can propose only a price at an integer grid when the tick size is d. That is,

$$\mathrm{P} \in \{0, d\}^{12} \tag{1}$$

The purpose of this paper is to model platform competition, and our model is parsimonious for limit and market orders. Traders do not choose the order type, the order book is empty when

[10] In reality, a market order can trade with a limit order on another exchange due to regulation NMS. However, there is a routing fee for cross-exchange execution.

[11] When the tick size is smaller than d but great greater than 0, the liquidity maker's quote P becomes a complex and discontinuous function of the fee structure. This adds to the mathematical complexity without conveying additional intuitions.

[12] In a more complex version of the model, a liquidity maker can propose $P \in \{k \cdot d\}_{k=-\infty}^{\infty}$, but the result is similar.

the liquidity maker arrives, and our three-stage model involves only one trading round. Therefore, our model does not allow for limit-order queuing. Theoretical studies on order-placing strategy generally provide a richer structure of order selection by assuming exogenous stock exchanges (Rosu (2009), Foucault and Menkveld (2008), Parlour (1998), and Parlour and Seppi (2003)). We show that endogenizing the decision of operators significantly complicates the game: the game between operators reaches complex mixed-strategy equilibria even granting the abovementioned simplifications. Nevertheless, our simple model explains several stylized facts that have not been addressed in the literature.

B Benchmark: Continuous Tick Size

Lemma 1 summarizes the market outcome of our model when the liquidity maker can propose any trading price P.

Lemma 1 (Neutrality of Fees and Fee Structure under a Continuous Tick Size)

Under a continuous tick size,

(i) The liquidity makers' strategy and the liquidity takers' strategy depend only on the total fee *but not its breakdown.*

(ii) Competing platforms belonging to independent operators choose a zero total fee exclusively and earn zero profits.

(iii) A monopoly operator has no incentive to open more than one platform for any positive fixed cost of opening a platform.

Proof: See the appendix

Part *(i)* and *(ii)* of Lemma 1 offer similar economic intuitions as Colliard and Foucault (2012). Part *(i)* follows the canonical tax-neutrality principle. It implies that the platforms compete on one dimension: the total fee. Holding total fee fixed, an increase (decrease) of the make fee decreases (increases) the buy limit-order price proposed by the liquidity maker by the same amount, leading the cum fee buy price unchanged. The liquidity maker always chooses the platform with the lowest total fee. Part *(ii)* shows that the competition on total fees between platforms owned by competing operators leads to Bertrand equilibrium, as the competing operators have incentives to undercut each other towards zero total fees. Part *(iii)* of Lemma 1 shows that the operator has no incentive to offer multiple platform under continuous tick size. If a monopoly operator establishes multiple platforms, the one with the lowest total fee attracts all traders, leaving no benefit for the operator to maintain other platforms.

Although the fee neutrality, Bertrand competition, and the absence of price discrimination are consistent with intuitions implied by canonical principles, they are not consistent with the stylized facts. Next, we consider the case in which the tick size equals d, and demonstrate how such a small friction can generate results that are dramatically different from those predicted by conventional wisdom, and yet be consistent with the reality.

II. Non-Neutrality

In a single platform model, Foucault, Kadan and Kandel (2013) show that fee is no longer neutral with discrete tick size. The case under competing platforms offers additional insights to this non-neutrality. We demonstrates that, for the same amount of total free, liquidity maker may prefer a market that charges her to a market that subsidizes her. This result surprising. In our game, the liquidity taker seems to play a passive role: she can trade only on the platform selected by the liquidity maker, because the unchosen platform has an empty limit-order book. It thus seems that the priority of a platform is to attract liquidity makers, and a natural way to encourage liquidity provision is to subsidize makers. Therefore, the traditional view of the rebate to liquidity makers is to provide incentives for liquidity provision (Malinova and Park (2015)). Yet this interpretation cannot fully account for the existence of the taker/maker market which charges liquidity maker. Foucault, Kadan, and Kandel (2013) demonstrate that a monopoly platform may choose to subsidize takers to maximize the trading rate of the platform. Our paper advances the intuition by showing that liquidity maker can prefer a taker/maker market to a maker/taker market.

Lemma 2 introduces two concepts for future analysis: cum fee buy and sell prices.

Lemma 2 (Fee Structure, Trading Price and Participation with One Platform)

With tick size constraints (1), given make-take fees $F^i = (f_m^i, f_t^i)$, *the following results hold.*

(i) In order for a trade to happen, the platform must charge one side while subsidizing the other side. Moreover, the total fee cannot exceed the tick size. That is,

$$f_m^i \cdot f_t^i < 0. \tag{2}$$

and

$$f_m^i + f_t^i \leq d. \tag{3}$$

(ii) Conditional on choosing platform i, *the liquidity maker will propose a buy price of*

$$P = \begin{cases} 0 & when\ f_m^i > 0\ (so\ that\ f_t^i < 0) \\ d & when\ f_m^i < 0\ (so\ that\ f_t^i > 0) \end{cases}. \tag{4}$$

Which leads to cum fee buy and sell prices of

$$\begin{aligned} p_b^i &\equiv P + f_m^i = \begin{cases} f_m^i & when\ f_m^i > 0 \\ d + f_m^i & when\ f_m^i < 0 \end{cases} \\ p_s^i &\equiv P - f_t^i = \begin{cases} -f_t^i & when\ f_m^i > 0 \\ d - f_t^i & when\ f_m^i < 0 \end{cases} \end{aligned} \tag{5}$$

Proof: *See the appendix.*

Equation (5) shows that the make-take fees of a platform *i* uniquely determine cum fee buy and sell prices. To create cum fee buy and sell price within the tick, the make and take fee must carry the opposite sign. A platform must charge one side while subsidizing the other side in order for a trade to happen. This result is related to our simplifying assumptions on the traders' valuation and price grid, but the prediction is consistent with the stylized facts. In reality, it is rare for major exchanges to charge both makers and takers. Cardella, Hao, and Kalcheva (2012) document 133 fee structure changes during 2008–2010 across major exchanges, and no platforms ever charge both sides in their sample.[13] To the best of knowledge, no existing literature provide an explanation on why make-take fees always carry opposite signs in their sample. Lemma 2 nevertheless provide the first explanation: when liquidity maker and taker's valuation is within the same tick, fee of opposite signs are able to create transaction price within the tick.

[13] We thank Laura Cardella for helping us confirm this claim.

The focus of this paper is on make-take fee, but our model is flexible enough to accommodate other efforts of the operator to create sub-penny transaction prices. One such effort is the creation of mid-point peg orders. These orders have a price equal to the midpoint of the best bid and offer. Because SEC 612 does not allow sub-penny displayed orders, these orders are usually hidden. In our model, a midpoint peg order has cum fee buy and sell prices of $\frac{1}{2}d$. Such prices can also be achieved through a fee structure of $\left(f_m^i = \frac{1}{2}d, f_t^i = -\frac{1}{2}d\right)$ or $\left(f_m^i = -\frac{1}{2}d, f_t^i = \frac{1}{2}d\right)$ in the absence of midpoint peg order.[14] Starting from next section, we focus the analysis on cum fee buy and sell prices. The main purpose of this paper is to interpret the proliferations of stock exchanges. Yet our paper can also rationalize the creation of new order types to bypass the tick size constraints.

Proposition 1 demonstrates the non-neutrality of fee breakdowns as well as the condition under which liquidity makers prefer being charged instead of being subsidized.

Proposition

With tick size constraints (1), suppose platform 1 adopts fee structure (f_m, f_t)*, and platform 2 adopts fee structure* (f_t, f_m)*, where* $d > f_m > -f_t > 0$*. All liquidity makers prefer platform 1 (or 2) when* $|f_m| + |f_t| < d$ *(or* $|f_m| + |f_t| > d$*), and they are indifferent between the two platforms only when* $|f_m| + |f_t| = d$*.*

Proof: *See the appendix.*

When $|f_m| + |f_t| \neq d$, Proposition 1 shows that fee breakdown is no longer neutral. Liquidity maker can prefer one platform to the other despite the same total free. More surprisingly, when the tick size is large relative to the level of the make-take fees, the liquidity maker prefers a market that charges her and subsidizes the liquidity taker. Fixing the level of make-take fees, as the tick size decreases, the liquidity maker shifts her preference to a market that subsidizes her and

[14] The exchanges often charges fees to mid-point peg orders, which leads to cum fee buy and sell price different from $\frac{1}{2}d$. These adjusted cum fee buy and sell price can be achieved using other fee structures.

charges the liquidity taker. The intuition behind this result can be obtained by comparing cum fee buy prices $p_b^1 = f_m$ and $p_b^2 = d + f_t$ for the liquidity maker. It is easy to verify that $p_b^1 < p_b^2$ when $f_m - f_t < d$, or $|f_m| + |f_t| < d$. Therefore, the liquidity maker's gains from trade are actually lower with a subsidy, and the formal proof of Proposition 1 shows that the corresponding increase in the execution probability is not large enough to offset the decrease in the gains from the trade. Therefore, the liquidity maker prefers a market that charges her when the absolute value of the fees is relatively small. An increase in the absolute value of the fee or a decrease in the tick size changes this relationship as $p_b^1 > p_b^2$ when $|f_m| + |f_t| > d$. In this case, the platform that subsidizes the liquidity maker provides a lower cum fee buy price, and the liquidity maker prefers a subsidy to a fee.

Proposition 1 provides a plausible justification for the existence of the taker/maker market. The emergence of markets which charge liquidity makers is a puzzle, particularly when regulations can put taker/maker markets at a disadvantage. One such policy is the trade-through rule.[15] To the best of our knowledge, there is no theoretical explanation of the comparative advantage of a market that charges liquidity makers when it competes with a market that subsidizes liquidity makers. Our paper fills this gap. The result is also consistent with the empirical evidence in Yao and Ye (2014) that taker/maker markets attract volume for securities with large relative tick size and maker/taker markets attract volume for securities with low relative tick size.

Proposition 1 states that the liquidity maker is indifferent between a fee and a subsidy when the sum of the absolute values of the fees is equal to one tick. This result is surprising because papers in the literature on two-sided markets generally predict that a specific side will be subsidized given the parameters of the environment. Our results differ because the liquidity maker can adjust her quotes based on the fee structure. Consider the following two fee structures

[15] In the United States, orders are routed to the market with the best nominal price. This regulation favors markets that subsidize makers. To see this, start with the model of Colliard and Foucault (2012). Their model predicts that the taker/maker market and the maker/taker market can co-exist when they have the same total fees. The taker/maker market has a wider nominal quoted spread and the maker/taker market has a narrower nominal quote spread, although the spread after the fee is the same. The trade-through rule, however, is imposed on the nominal price, which implies that the taker/maker market cannot win the competition with the maker/taker market because the latter has a better nominal price, ceteris paribus.

$F_1 = (f_m, f_t)$ and F_2= (f_t, f_m) with $d > f_m > -f_t > 0$. The liquidity maker proposes a buy price of 0 under fee structure F_1, which leads to a cum fee buy price of $p_b^1 = f_m$ and a cum fee sell price of $p_s^1 = -f_t$. The liquidity maker has to propose a buy price of d under fee structure F_2, which results in a cum fee buy price of $p_b^2 = d + f_t$ and a cum fee sell price of $p_s^2 = d - f_m$. It is easy to verify that these two fee structures lead to the same cum fee buy and sell prices when $|f_m| + |f_t| = d$. Another way to understand the result is that $|f_m| + |f_t| = d$ implies $f_m = d + f_t$, so $F_1 = (d + f_t, f_t)$ and $F_2 = (f_t, d + f_t)$. Therefore, 1) the two fee structures have the same total fee and 2) the make fee on platform 1 is one tick higher and the take fee on platform 1 is one tick lower than those on platform 2. Proposition 1 implies that the liquidity maker can neutralize the fee change in the multiple of the ticks. The Lemma 1 can be considered as a limit case of Proposition 1. When tick size is continuous, liquidity maker can neutralize any fee breakdowns.

III. Product Differentiation and Liquidity Makers' Segmentation

This section demonstrates that the non-neutrality of the fee structure, led by the tick size constraints, allows operators to create vertical differentiation for otherwise identical platforms. The previous section shows that the nature of fee competition is to choose cum fee buy and sell prices $(p_b^{\mathrm{i}}, p_s^{\mathrm{i}})$. In the following analysis, we consider each platform's decision variables as cum fee buy and sell prices to avoid a tedious discussion of fee structures that achieve the same equilibrium outcome.

A. Vertical Product Differentiation

Given the cum fee sell price, the marginal liquidity taker's valuation on platform *i* is given by

$$\hat{v}_s^i \equiv \min\left\{p_s^{\mathrm{i}}, \frac{d}{2}\right\}. \tag{6}$$

So the probability that a liquidity taker accepts the buy limit order is given by

$$q_s^{\mathrm{i}} = \Pr\left(v_s \le \hat{v}_s^i\right) = \frac{2}{d} \cdot \hat{v}_s^i \tag{7}$$

Therefore, a higher cum fee sell price implies a higher probability of execution.

The liquidity maker's surplus for choosing platform *i* is

$$BS^{\mathrm{i}} = \frac{2}{d} \cdot \left(v_b - p_b^{\mathrm{i}}\right) \cdot \Pr\left(v_s \le \hat{v}_s^i\right) = \frac{2}{d} \cdot \left(v_b - p_b^{\mathrm{i}}\right) \cdot \min\left\{p_s^{\mathrm{i}}, \frac{d}{2}\right\} \tag{8}$$

Equation (8) shows that the liquidity maker's surplus increases with cum fee sell price when $p_s^{\mathrm{i}} \le \frac{d}{2}$, because a higher cum fee sell price increases a liquidity maker's execution probability. Therefore, other things remaining equal, the liquidity maker prefers a platform with a higher cum fee sell price. From a liquidity maker's point of view, platforms with differentiated cum fee sell prices are vertically differentiated: the platform with the higher cum fee sell price has higher quality in the sense that orders on this platform have higher execution probabilities. Such product differentiation is the fundamental rationale behind the second-degree price discrimination in section IV and the non-Bertrand outcome in section V.

Notice that the operator's choice of price and quality differs from what occurs in a typical price-quality game. From the liquidity maker's point of view, a platform chooses a price of execution services (the make fee) and the quality of execution services (the execution probability implied by thecum fee sell price) simultaneously, whereas a firm in a typical price-quality game chooses quality first and then the price. It is well known that the sequential move from quality to price destroys Bertrand equilibrium, but non-Bertrand pure-strategy equilibrium still exists (Shaked and Sutton (1982)). Section V shows that the simultaneous choice of price and quality in our model destroys not only Bertrand equilibrium, but also any pure-strategy equilibrium.

This discussion shows that the market outcome depends critically on the ability of end users to neutralize the fee. We illustrate the result using the stock exchange industry, but we believe the intuition should hold in other contexts as well. If end users can neutralize the fee that is set by an operator, the competition between platforms is only one-dimensional. If end users cannot neutralize the fee, then an operator has more power and flexibility for manipulating the fee structure. The two-sided platforms are able to create product differentiation that is not otherwise available in a one-sided market due to the non-neutrality of the fee structure.

Next, we show that such product differentiation leads to market fragmentation. This provides a formal justification for the observation that "(i)t is relatively uncommon for industries based on two-sided platforms to be monopolies or near monopolies" (Evans and Schmalensee (2007), p. 166). The fragmentation of two-sided markets is a puzzle, because work in the two-sided market literature is overwhelmingly based on network effects (Rysman (2009)), and network effects in general tend to induce consolidation. To the best of our knowledge, our paper is the first to show that two-sidedness may add an extra dimension that facilitates product differentiation and hence market fragmentation.

B. Liquidity Makers' Segmentation under Two Platforms

While there is a consensus among liquidity makers that a platform with a higher cum fee sell price is of higher quality, liquidity makers differ in their willingness to pay for quality. In this subsection, we look closely at the segmentation of liquidity makers between two vertically differentiated platforms.

Given the cum fee buy and sell prices on platforms 1 and 2, (p_b^1, p_s^1) and (p_b^2, p_s^2), the liquidity maker's surpluses when choosing platform 1 and platform 2 are

$$\begin{aligned} BS^1 &= \frac{d}{2} \cdot (v_b - p_b^1) \cdot \hat{v}_s^1 = \frac{d}{2} \cdot (v_b - p_b^1) \cdot p_s^1 \\ BS^2 &= \frac{d}{2} \cdot (v_b - p_b^2) \cdot \hat{v}_s^2 = \frac{d}{2} \cdot (v_b - p_b^2) \cdot p_s^2 \end{aligned} \tag{9}$$

The equalities above follow $\hat{v}_s^i = p_s^i$ $(i = 1,2)$. This is because neither platform would set $p_s^i > \frac{d}{2}$ so that $\hat{v}_s^i = \frac{d}{2}$ $(i = 1,2)$, as doing so would reduce its per-trade profit while not gaining any trading volume.

When $p_s^1 = p_s^2$, $BS^1 \gtreqless BS^2$ if and only if $p_b^1 \lesseqgtr p_b^2$. Without loss of generality, suppose that $p_s^1 < p_s^2$, which implies that platform 1 has lower execution probability and platform 2 has higher execution probability. The liquidity maker's surpluses under the two platforms are shown in Figure 2.

Insert Figure 2 about Here

When $p_b^1 \geq p_b^2$, as shown in panel (a) of Figure 2, $BS^1 \leq BS^2$ for any $v_b \geq p_b^2$. So all liquidity makers choose platform 2, because platform 2 offers higher execution probability along with a lower cum fee buy price.

When $\mathrm{p}_\mathrm{b}^1 < \mathrm{p}_\mathrm{b}^2$, as shown in panels (b) and (c) of Figure 3, there exists a unique intersection

$$\varphi \equiv p_b^1 + \left(p_b^2 - p_b^1\right) \cdot \frac{p_s^2}{p_s^2 - p_s^1} \tag{10}$$

and $BS^1 \lesseqgtr BS^2$ for any $v_b \gtreqless \varphi$. Recall that $v_b \in \left[\frac{d}{2}, d\right]$; as we can show $\varphi > \frac{d}{2}$, all that remains is to check whether $\varphi > d$. The boundary of $\varphi = d$ in (p_b^2, p_s^2)-plane is given by

$$p_s^2 = \rho(p_b^2) \equiv p_s^1 \cdot \frac{d - p_b^1}{d - p_b^2} \tag{11}$$

or equivalently,

$$p_b^2 = \omega(p_s^2) \equiv d - p_s^1 \cdot \frac{d - p_b^1}{p_s^2} \tag{12}$$

When $\varphi > d$, or $p_b^2 > d - (d - p_b^1) \cdot \frac{p_s^1}{p_s^2}$, all liquidity makers choose the platform with lower execution probability, as shown in panel (c) of Figure 2, because the price of the high-quality platform is too high to justify its higher execution probability.

Panel (b) of Figure 2 demonstrates the most interesting case with $\varphi \leq d$. Under this scenario, the platform with higher execution probability and the platform with lower execution probability co-exist. This happens when platform 2 sets a higher cum fee buy price than the platform 1 does, but the cum fee buy price on platform 2 is not high enough to drive the liquidity maker with the highest gains from trade to platform 1. The fragmentation of the market arises from the heterogeneity of liquidity makers' valuations. Ceteris paribus, all liquidity makers prefer a platform offering higher execution probability. Yet makers and takers are not equally inclined to choose the higher execution probability. Liquidity makers with larger gains from trade care more about execution probability than do liquidity makers with relatively smaller gains from trade. The heterogeneity of valuations across traders allows the vertically differentiated platforms to charge different prices for different execution probabilities on each platform.

The makers' choices given (p_b^1, p_s^1) and (p_b^2, p_s^2) are summarized in the lemma 3 and Figure 3.

Lemma 3 (Liquidity Makers' Segmentation under Two Platforms)

For any given $(p_b^1, p_s^1) \in \left[\frac{d}{2}, d\right] \times [0, \frac{d}{2}]$, *the square* $\left[\frac{d}{2}, d\right] \times [0, \frac{d}{2}]$ *in the* (p_b^2, p_s^2)*-plane can be divided into the following six areas:*

(i) $s_1 \equiv \{(p_b^2, p_s^2) \left| \frac{d}{2} \leq p_b^2 \leq p_b^1, p_s^1 \leq p_s^2 \leq \frac{d}{2}\right.\}$*: no liquidity maker chooses platform 1, and all liquidity makers with* $p_b^2 \leq v_b \leq d$ *choose platform 2;*

(ii) $s_2 \equiv \{(p_b^2, p_s^2) | p_b^1 \leq p_b^2 \leq \omega(p_s^2), p_s^1 \leq p_s^2 \leq \frac{d}{2}\}$*: liquidity makers with* $p_b^1 \leq v_b \leq \varphi$ *choose platform 1, and liquidity makers with* $\varphi \leq v_b \leq d$ *choose platform 2;*

(iii) $s_3 \equiv \{(p_b^2, p_s^2) | p_b^1 \leq p_b^2 \leq d, p_s^1 \leq p_s^2 \leq \min\left\{\rho(p_b^2), \frac{d}{2}\right\}\}$*: all liquidity makers with* $p_b^1 \leq v_b \leq d$ *choose platform 1, and no liquidity maker chooses platform 2;*

(iv) $s_4 \equiv \{(p_b^2, p_s^2) | p_b^1 \leq p_b^2 \leq d,\ 0 \leq p_s^2 \leq p_s^1\}$*: all liquidity makers with* $p_b^1 \leq v_b \leq d$ *choose platform 1, and no liquidity maker chooses platform 2;*

(v) $s_5 \equiv \{(p_b^2, p_s^2) \left| \frac{d}{2} \leq p_b^2 \leq p_b^1, 0 \leq p_s^2 \leq \rho(p_b^2)\right.\}$*: liquidity makers with* $\varphi \leq v_b \leq d$ *choose platform 1, and liquidity makers with* $p_b^2 \leq v_b \leq \varphi$ *choose platform 2;*

(vi) $s_6 \equiv \{(p_b^2, p_s^2) \left| \frac{d}{2} \leq p_b^2 \leq p_b^1, \rho(p_b^2) \leq p_s^2 \leq p_s^1\right.\}$*: no liquidity maker chooses platform 1, and all liquidity makers with* $p_b^2 \leq v_b \leq d$ *choose platform 2.*

Here φ, $\rho(p_b^2)$, *and* $\omega(p_s^2)$ *are given by (10), (11), and (12), respectively.*

Proof: *See the appendix.*

Insert Figure 3 about Here

For any given (p_b^1, p_s^1), the square $\left[\frac{d}{2}, d\right] \times \left[0, \frac{d}{2}\right]$ in the (p_b^2, p_s^2)-plane can be divided into six areas. In area s_1, platform 2 attracts all liquidity makers by offering higher execution probability with a lower cum fee buy price, as $p_s^2 > p_s^1$ and $p_b^2 < p_b^1$. This area corresponds to panel (a) of Figure 3. In area s_2, both platforms co-exist so the market is fragmented. Platform 2 attracts liquidity makers with larger gains from trade because the execution probability is higher, and platform 1 appeals to liquidity makers with smaller gains from trade. This area corresponds to panel (b) of Figure 3. In area s_3, the curve $p_s^2 = \rho(p_b^2)$ describes the case in which the maker with the highest gains from trade is indifferent between the two platforms. This area corresponds to panel (c) of Figure 3. In this case, the cum fee buy price on platform 1 is so low compared with that on platform 2 that even the liquidity maker with the highest gain from trade prefers platform 1. The interpretations for areas $s_4, s_5,$ and s_6 follow the same logic as $s_1, s_{2,}$ and s_3.

Our results pertaining to the segmentation of liquidity makers explain the puzzle raised by Skjeltorp, Sojli, and Tham (2011), who find evidence that NASDAQ and NASDAQ BX, two platforms operated by the NASDAQ OMX group, attract different clients.[16] Foucault (2012) suggests that the co-existence of various make/take fees should serve to screen investors by type. However, Foucault (2012) also mentions that "it is not clear however how the differentiation of make/take fees suffices to screen different types of investors since, in contrast to payments for order flow, liquidity rebates are usually not contingent on investors' characteristics (e.g., whether the investor is a retail investor or an institution)." This paper explains this puzzle: when end users cannot neutralize the breakdown of the fee, and the markets therefore become two-sided, the operators can screen liquidity makers based on the terms of the trade offered to the liquidity takers. This explains, as we show in the next section, why operators have an incentive to open multiple platforms to price-discriminate against traders.

[16] They find that NASDAQ BX might be used by algorithmic investors who use algorithms to minimize execution costs (agency algorithms) rather than to quickly exploit private information.

IV. Price Discrimination

This section endogenizes a monopoly operator's decision regarding the number of platforms to offer and the fee structure on each platform. The purpose is to explore the second-degree price discrimination facilitated by product differentiation. Subsection IV A first considers the benchmark case in which the monopoly operator establishes one platform, and Subsection IV B then shows the economic incentive for the monopoly operator to open more than one platform on which to practice price discrimination in.

A. Benchmark: One Operator with One Platform

The monopoly operator with one platform chooses (p_b, p_s) to maximize its profit

$$\pi = (p_b - p_s) \cdot q_b \cdot q_s$$
$$= \frac{4}{d^2} \cdot (p_b - p_s) \cdot (d - \hat{v}_b) \cdot \hat{v}_s,$$
$$= \frac{4}{d^2} \cdot (p_b - p_s) \cdot (d - p_b) \cdot p_s$$

where the first equality follows from equation (6). Since π increases with p_b (or decreases with p_s) whenever $\hat{v}_b = \max\left\{p_b, \frac{d}{2}\right\} = d/2$ (or $\hat{v}_b = \min\left\{p_s, \frac{d}{2}\right\} = d/2$), it is easy to see that the monopoly operator will always set (p_b, p_s) such that $\max\left\{p_b, \frac{d}{2}\right\} = p_b$ and $\min\left\{p_s, \frac{d}{2}\right\} = p_s$. So the second equality follows.

The key trade-off involved for the operator that sets the fee structure is the profit conditional on execution and *both* sides' participation probabilities. It is straightforward to solve this optimization problem, and the equilibrium outcomes are summarized in Lemma 4.

Lemma 4 (Optimal Monopoly Fees and Equilibrium Surplus Divisions)

With tick size constraints (8), the monopoly operator sets its cum fee buy and sell prices as

$$p_b^M = \frac{2}{3} \cdot d, \qquad p_s^M = \frac{1}{3} \cdot d. \tag{13}$$

The liquidity makers' surplus, the liquidity takers' surplus, and profit for the operator are

$$BS^M = \frac{2}{27} \cdot d, \qquad SS^M = \frac{2}{27} \cdot d, \qquad \pi^M = \frac{4}{27} \cdot d. \tag{14}$$

Both fee structures set by the monopoly operator impose a cum fee buy price of $\frac{2}{3}d$ and a cum fee sell price of $\frac{1}{3}d$. The operator obtains $\frac{1}{3}d$ once a trade happens, and 0 otherwise. This fee structure excludes liquidity makers and liquidity takers with low gains from trade, each of which comprises one-third of the liquidity makers' and liquidity takers' populations. By excluding liquidity makers and liquidity takers from whom the operator profits less, the operator attracts only liquidity makers and liquidity takers with high gains from trade and enjoys monopoly profits.

The operator's fee choice in our model entails two dimensions: the total fee and the breakdown of the total fee. This two-dimensional optimization differentiates our study from two existing studies on make-take fees. Colliard and Foucault (2012) study the total fee in an environment where the breakdown is neutral, and Foucault, Kadan, and Kandel (2013) address the breakdown of the fees given a fixed total fee. As we shall see, such two-dimensional optimization generates dramatically different predictions from those found in the existing literature.

B. Price Discrimination with Multiple Platforms

In this subsection, we explain the incentive that induces an operator to run multiple platforms. We introduce a new mechanism to the price-discrimination literature: when end users cannot neutralize the fee structure, the operator can use fees charged to the liquidity takers and their implied execution probability to price-discriminate against the liquidity makers. With differentiated execution probability implied by heterogeneous cum fee sell prices, liquidity makers self-select based on their expected surpluses from each platform. This mechanism corresponds to *second-degree* price discrimination, which is different from the mechanism of payments for order flow, a common third-degree price discrimination under which traders are charged differently based on their identities (retail or institutional).

Proposition 2 (Number of Platforms Established by a Monopoly Operator)

Suppose that a monopoly operator is allowed to operate k platforms; the optimal cum fee buy and cum fee sell prices in each platform i are

$$p_b^i = \frac{d}{2} + \frac{i \cdot d}{2(2k+1)}, p_s^i = \frac{i \cdot d}{2k+1} \quad with\ 1 \leq i \ \leq k. \tag{15}$$

The liquidity makers' surplus, liquidity takers' surplus, and the monopoly operator's profit are

$$BS^M(k) = \frac{k \cdot (k+1)}{3(2k+1)^2} \cdot d, \qquad SS^M(k) = \frac{k \cdot (k+1)}{3(2k+1)^2} \cdot d, \qquad \pi^M(k) = \frac{2k \cdot (k+1)}{3(2k+1)^2} \cdot d, \tag{16}$$

respectively, which all increase in k.

Suppose that opening a new platform requires a fixed cost c; the number of platforms opened by the monopoly operators is

$$\bar{k} = \max\left\{k \in \mathbb{N} \middle| \frac{4k \cdot d}{3(2k+1)^2(2k-1)^2} \geq c\right\}.$$

Proof: *See the appendix.*

Proposition 2 shows that the operator's profit always increases with the number of platforms established. Therefore, the operator has incentives to establish infinitely many platforms in the absence of a fixed cost. A fixed cost constrains the number of platforms, as the marginal benefit of adding one platform decreases as the number of existing platforms rises. Surprisingly, not only the operator's profit, but also the liquidity makers' and liquidity takers' surpluses, increase with the number of platforms. The welfare gain originates from a higher rate of participation: increasing the number of platforms creates more cum fee price levels within the tick. For example, the lowest cum fee buy price across all platforms of the liquidity maker, $\frac{k+1}{2k+1} \cdot d$, increases with the total number of platforms *k* and approaches $\frac{d}{2}$ as *k* goes to infinity. The liquidity maker submits one limit order almost surely with infinitely many platforms. As the inefficiency originates from the discrete tick size, the creation of new cum fee price levels by setting up new platforms reduces this inefficiency and generates gains from trading for all parties.

The nature of the second-degree price discrimination can be illustrated by considering the example with two platforms. Consider Equation (15) under the case $k = 2$. When a monopoly operator is allowed to open two platforms, she establishes one platform with low cum fee sell price $p_s^1 = \frac{1}{5} \cdot d$ and the other platform with cum fee sell price $p_s^2 = \frac{2}{5} \cdot d$. The liquidity maker would consider platform 1 to be of low quality because her order on platform 1 has execution probability of $\frac{2}{5}$ ($\Pr(v_s \leq \frac{1}{5} d)$), and would consider platform 2 to be of high quality because her order on platform 2 has execution probability of $\frac{4}{5}$. The operator, however, charges a lower cum fee buy price of $p_b^1 = \frac{3}{5} \cdot d$ on the lower-quality platform 1 and a higher cum fee buy price of $p_b^2 = \frac{7}{10} \cdot d$ on the higher-quality platform 2. Figure 4 illustrates the diagram of two such price–execution probability packages which induce liquidity makers to self-select: liquidity makers with valuations between $[\frac{3}{5} d, \frac{4}{5} d]$ select platform 1 and liquidity makers with valuations between $[\frac{4}{5} d, d]$ select platform 2.

Insert Figure 4 About Here

V. Competing Operators and the Non-existence of Pure-strategy Equilibrium

Now we consider the case with two competing operators, each of which establishes one platform. The model yields complex and interesting outcomes even with this simplification. Section V A shows the non-existence of pure-strategy equilibrium under the tick size constraints. Section V B shows that symmetric mixed-strategy equilibria, in which both exchanges earn strictly positive profits, always exist.

A. No Pure-strategy Equilibrium

The fact that the tick size constraints destroy Bertrand equilibrium can be understood intuitively based on product differentiation. Operators can create platforms with diverse execution probabilities, which alleviate the competitive pressure on otherwise identical platforms. The surprising result is that the tick size constraints also destroy *any* pure-strategy equilibrium, which is summarized in Proposition 3.

Proposition 3 (No Pure-strategy Equilibrium)

There is no pure-strategy equilibrium when two platforms compete under tick size d.

Proof: *See the appendix.*

The detailed proof of the proposition is in the appendix. We offer here a sketch of the proof and the corresponding intuitions. We first prove the non-existence of pure-strategy equilibrium with any platform earning a positive profit, which follows from the Bertrand argument in Colliard and Foucault (2012). Without loss of generality, suppose platform 1 earns a strictly positive profit and platform 2 begins by earning a lower profit or having the same profit as platform 1. In the former case, platform 2 can always increase its profit by undercutting platform 1's cum fee buy price by ε and mimicking its cum fee sell price. By doing so, platform 2 offers the same execution probability but with a lower cum fee buy price. Thus, all liquidity makers choose platform 2, and platform 2 will earn what platform 1 earned before. In the latter case, by the same undercutting strategy, platform 2 can corner the entire market, rather than sharing the market with platform 1, with only ε concession per trade. Therefore, there is no pure strategy equilibrium with any platform earning positive profits.

The standard Bertrand argument seems to suggest that both platforms should end up with zero profits and zero fees. However, we find that, due to the two-sidedness of the markets as well as the heterogeneity of liquidity maker/taker valuations, one platform can always find a profitable deviation strategy if the other platform maintains a pure strategy.

There exist two possible scenarios which give rise to the zero-profit outcome: 1) at least one side of the market does not participate; 2) the cum fee buy and sell prices are equal. It is easy to see that the first case cannot be sustained in equilibrium, because one of the platforms would have incentives to facilitate some trading and profit from it. We next examine the scenario in which the cum fee buy price equals the cum fee sell price. Three cases are to be considered.

First, consider the case in which platform 1 charges $p_b^1 = p_s^1 > \frac{1}{2} \cdot d$. Then platform 2 can easily deviate by setting $p_b^2 = p_b^1 - \varepsilon$ and $p_s^2 = \frac{1}{2} \cdot d$. In this case platform 2 reduces the cum fee sell price but does not reduce the execution probability for liquidity makers because all liquidity takers accept $p_s^2 = \frac{1}{2} \cdot d$. All liquidity makers thus choose platform 2 as it offers the same execution probability as platform 1, but they enjoy higher gains from trade conditional on execution.

Second, consider the case in which platform 1 charges $p_b^1 = p_s^1 = \frac{1}{2} \cdot d$. Panel (a) of Figure 5 demonstrates one deviating strategy for platform 2, which sets $p_b^2 = \frac{1}{2} d - \mu\varepsilon$, and $p_s^2 = \frac{1}{2} d - \varepsilon$ with $\varepsilon > 0$ and $0 < \mu < 1$. This deviation reduces the execution probability, which leads to a flatter liquidity maker surplus function on platform 2. However, the intersection of BS_2 with the horizontal axis (p_b^2) falls to the left of the intersection of BS_1 with the horizontal axis (p_b^1), which implies that BS_2 crosses BS_1 at the point where $\underline{v_b} > \frac{1}{2} \cdot d$. Liquidity makers with valuations between $[\frac{1}{2} \cdot d, \underline{v_b})$ then prefer platform 2 to platform 1, and thus platform 2 can enjoy strictly positive profits.[17] The intuition is as follows: even when platform 1 provides the maximum execution quality and charges the lowest price for sustaining that quality, platform 2 can deviate by charging an even lower price while making only an infinitesimal sacrifice in execution probability. This strategy caters to makers with low gains from trade, such as those with valuations close to $\frac{1}{2} \cdot d$.

Insert Figure 5 about Here

Last, consider the case in which platform 1 charges $p_b^1 = p_s^1 < \frac{1}{2} \cdot d$, for which the execution probability is less than 1. Then platform 2 can deviate by setting $p_b^2 = p_b^1 + \varepsilon$ and $p_s^2 = p_s^1 + \mu \cdot \varepsilon$, with $\varepsilon > 0$ *and* $0 < \mu < 1$. Such a deviation involves a higher cum fee buy price for liquidity makers but they are compensated with a higher probability of execution for their orders. Panel (b) of Figure 5 illustrates that this deviation makes BS_2 steeper than BS_1, although the

[17] $\underline{v_b}$ should be smaller than d when ε is small.

intersection of BS_2 with the horizontal axis falls to the right of BS_1. It is clear that there exists a point $\overline{v_b} < d$ such that liquidity makers with valuations higher than $\overline{v_b}$ go to platform 2.

One important driver of the non-existence of pure-strategy equilibrium comes from the simultaneous determination of the price of execution services through the make fee and the "quality" of execution services through the take fee.[18] We find that if we allow the operator to choose the take fee in the first stage and the make fee in the second stage, the model would also have the usual non-Bertrand pure-strategy equilibrium under sequential choice of quality and then price (unreported for briefness). In most industries, it is natural to assume that the quality of a product is determined before setting the price, because product quality is a long-term decision while price is a short-term decision. Yet in our model, the "quality" in terms of execution probability is essentially a pricing decision, as the "quality" is purely determined by the cum fee sell price. Such a cum fee sell price can be adjusted as easily as the cum fee buy price can. Therefore, it is reasonable to consider simultaneous price and quality competition here, rather than sequential moves as in the standard vertical differentiation literature.

More interestingly, the one-to-one mapping between the "quality" of a platform and its cum fee sell price points out that one way to envision two-sided market pricing is to consider its price on one side as a quality measure valued by the other side of the market, and that the platforms essentially choose price and quality simultaneously. We believe this link between two-sided markets in terms of pricing and vertical differentiation will be promising for future research.

The non-existence of pure-strategy equilibrium further motivates us to investigate, in the next subsection, the random nature of competing fee structures.

B. Mixed-strategy Equilibrium

This subsection focuses on characterizing symmetric mixed-strategy equilibria, in which both exchanges follow the same randomization when deciding their cum fee buy and cum fee sell prices (p_b, p_s).

[18] A recent paper by Chioveanu (2012) finds that simultaneous price and quality competition generates mixed-strategy equilibrium. Our paper finds a market that justifies such a simultaneous choice.

Proposition 4 (Mixed-strategy Equilibrium)

(i) *There exist symmetric mixed-strategy equilibria, in which (p_b, p_s) has a convex support on $[\frac{d}{2}, d] \times [0, \frac{d}{2}]$;*

(ii) *In equilibrium, both platforms earn strictly positive profits.*

(iii) *In any of the symmetric equilibria,*

$$\frac{p_s}{d} = g\left(\frac{p_b}{d}\right),$$

where $g(\cdot)$ is an increasing function.

Proof: *See the appendix.*

Proposition 4 proves the existence of mixed-strategy equilibria and describes their properties. The mixed strategies have a convex support, which implies that there is a connected range of cum fee buy and cum fee sell price pairs in which no specific pair is either better than or inferior to any of its neighbors. This result demonstrates the non-existence of an ideal fee structure that all the platforms should adopt, even in a probability sense. At first glance, liquidity makers and liquidity takers should all prefer the market that offers them the highest rebate, and it is puzzling why some platforms can survive with neither the highest rebate for liquidity makers nor the highest rebate for liquidity takers. Proposition 4 provides a plausible explanation of the diverse fee structures across platforms.

Part (ii) states that profits under mixed-strategy equilibrium are strictly positive, which could induce new platform entries and cause market fragmentation. This result arises from the two-sidedness of the markets caused by the tick size regulation. When the tick size is zero, as shown in Section I, the markets are one-sided. Hence, the competition between two platforms can drive profits to zero (Colliard and Foucault (2012)), which implies that any positive cost involved in establishing a new trading platform would deter entry. In reality, however, we continue to witness entries of new trading platforms. When the tick size is positive, the markets become two-sided. So the competition between platforms does not lead to zero profit for the platforms, which encourages new entries. Regulators are often concerned that the entry of new trading platforms generates greater market fragmentation (O'Hara and Ye (2011)), but the literature has achieved only a limited understanding of why the market becomes increasingly fragmented. We show that one force causing such fragmentation is the existing tick size regulation.

Part (iii) says that the support of any symmetric mixed-strategy equilibrium must be an upward-sloping curve that pushes the cum fee buy price higher than the cum fee sell price. This also confirms that competing platforms must earn strictly positive profits when they randomize their fees.

In the appendix, we show that mixed-strategy equilibria are in general characterized by two partial differentiation equations along with some boundary conditions. It is a daunting task to find analytical solutions for all possible mixed-strategy equilibria. One set of randomized pricing strategies is given in Corollary 1. Note that there might be other symmetric or asymmetric mixed-strategy equilibria.[19]

Corollary 1 (One Set of Symmetric Mixed-strategy Equilibria)

One set of symmetric mixed-strategy equilibria is as follows.

(a) $p_b - p_s = \frac{1}{6} \cdot d;$

(b) $p_b = x \cdot d$ *is randomized over* $[l, u] \equiv \left[\frac{1}{2} \cdot d,\ \frac{7}{12} \cdot d\right]$. x *has a cumulative distribution function* $F(x)$, *where*

$$F(x) = \frac{C_1}{\left(\frac{4}{3} - 3x\right)^{\frac{1}{3}}} + \frac{C_2}{\frac{5}{6} \cdot (x - \frac{1}{6})} \cdot (H - 1) \tag{17}$$

Here H *is a Hypergeometric function* ${}_2F_1\left(1, 1, \frac{5}{3}, \frac{5}{3 \cdot (6x - 1)}\right)$, *and* (C_1, C_2) *satisfy:*

$$\int_x^U t \cdot dF(t) - \int_L^x t \cdot dF(t) + \left(\frac{3}{2} - 2x\right) \cdot F(x) + \left(x - \frac{1}{6}\right)\left(\frac{4}{3} - 3x\right) \cdot F'(x) = \frac{1}{6} \tag{18}$$

(19)

[19] For example, we find that mixed strategy equilibria also exist when $p_b - p_s = \frac{1}{7} d$ or $\frac{1}{9} d$.

$$F'(x) > 0 \; for \; any \; x \in \left[\frac{1}{2}, \frac{7}{12}\right].$$

Proof: *See the appendix.*

VI. Empirical Results

The theoretical part of this paper predicts that a discrete tick size fragments the market by comparing the market outcome under a continuous tick size and with that under a discrete tick size d. Certainly, no securities are traded in continuous tick size in reality. Yet the relative tick size, defined as the uniform one-penny tick size divided by price, resembles a continuous tick size to a larger extent for high-priced securities than for low-priced securities. This section aims to show that a larger relative tick size causes more fragmented stock trading. Because our test is based on the cross-sectional variation in the relative tick size, the results need to be carefully interpreted. When an operator establishes an exchange, it is hard for the operator to consider its revenue stock by stock. Therefore, although we believe that the tick size drives market fragmentation, our cross-sectional choice should be more closely related to the choices made by liquidity makers and takers in our model. When the relative tick size is larger, liquidity makers and takers find it harder to neutralize the breakdown of the fees, which generates higher level of fragmentation for low-priced stocks. A small relative tick size facilitates neutralization of the fee breakdown and encourages consolidation. Section VI.A. describes the data used in testing this prediction; Section VI.B. presents the test results using multivariate regression analysis, and Section VI.C. tests this hypothesis using difference-in-differences analysis following the identification strategy proposed by Yao and Ye (2015).

A. Data and Sample

The empirical analyses use two securities samples from January 2010 through November 2011. The multivariate regression in Section VI.B. uses a sample of stocks selected by Hendershott and Riordan. The original sample includes 60 NYSE–listed and 60 NASDAQ–listed stocks. The stratified sample includes 40 large stocks from the 1000 largest Russell 3000 stocks, 40 medium stocks ranked from 1001–2000, and 40 small stocks ranked from 2001–3000. During our sample period, three of the 120 stocks were delisted (BARE, CHTT and KTII), our sample is thus consists of 117 stocks. The summary statistics for the stock and leveraged ETF samples are presented in Panel A of Table 1.

Section VI.C. tests the causal impact of the relative tick size on market fragmentation using a difference-in-differences approach. The identification follows Yao and Ye (2015), which use the split/reverse splits of leveraged ETFs as shocks to the relative tick size. The test uses leveraged ETFs that have undergone splits/reverse splits as the pilot group, and uses leveraged ETFs that track the same indexes and undergo no splits/reverse splits in our sample period as the control group. Leveraged ETFs amplify the return on the underlying index, and they often appear in pairs that track the same index but in opposite directions. For example, if the leverage ratio is 2:1 and if on one day the underlying index returns 1%, one ETF in the pair will return 2%, and the other one in the pair will return -2%.[20] Although twin leveraged ETFs often have similar nominal prices when launched for IPOs, the return amplification often diverges their nominal prices after issuance. As the ETFs are commonly issued by the same issuer, the issuers often use splits/reverse splits to keep their nominal prices aligned with each other. We use the ETF database and the Bloomberg Database to collect information on leveraged ETF pairs, and select the pairs that track the same index with an identical multiplier. The data are then merged with the CRSP to identify their reverse splitting events.

The variable of interest, market fragmentation, is constructed using TAQ data. The consolidated trade files of daily TAQ data provide information on executions across separate exchanges for trades greater than or equal to 100 shares (O'Hara, Yao, and Ye, 2014). We use the Herfindahl index as a measure of market fragmentation, which is calculated as follows:

$$Herfindahl\ Index_{i,t} = \sum_{j=1}^{13} \left(\frac{ExchVol_{j,i,t}}{TotalVol_{i,t}}\right)^2 \quad (20)$$

[20] The actual return will be slightly different, as management fees and transaction are yet to be taken into account.

where i indexes stock and t indexes time. $ExchVol_j$ denotes the trading volume on exchange j, $TotalVol$ while represents the total trading volume on all stock exchanges.[21]

The market fragmentation measure is then merged with the sample of 117 stocks and the leveraged ETF sample, respectively. The final sample used in the multivariate regression consists of 117 stocks in 51,950 stock-day observations. The final sample used in the difference-in-differences analysis consists of 5 splits and 23 reverse splits of leveraged ETFs from January 2010 through November 2011.[22] The sample window is 5 days before the reverse split event and 5 days after the reverse split event for the treatment and control groups. The summary statistics for the leveraged ETF sample are presented in Panel B of Table 1.

Insert Table 1 about Here

B. Regression Analysis

The key challenge to establish causal impact of relative tick size on HFT liquidity provision lies in addressing possible endogeneity issues (Roberts and Whited (2012)). One type of endogeneity arises from omitted variables. The estimation coefficient of the relative tick size would be biased and inconsistent if we did not control for variables that are correlated with both the nominal price and market fragmentation. A necessary condition for the occurrence of omitted variable bias is, therefore, that the omitted variable needs to correlate with nominal price. A recent paper by Benartzi et al (2009) finds that few variables can explain the cross-sectional variation in nominal price. Particularly, Benartzi et al (2009) do not find that firms actively manage their nominal price to achieve optimal relative tick size. If firms could choose their optimal relative tick sizes, they would aggressively split their stocks when the tick size changes from 1/8 to 1/16 and then to one cent. Such aggressive splits have not occurred in reality. Benartzi et al (2009) also reject several other hypotheses to explain nominal price, and then conclude with an explanation based on customs and norms with only two explanatory variables: market cap and industry. This

[21] We exclude the volume in TRFs because they have different trading mechanism.

[22] To ensure sufficient trading volume in these ETFs, we use leveraged ETFs that experience at least 10,000 share trading volume each day in the sample period.

finding facilitates our search for variables to control for omitted variable bias. The main specification in this paper controls for market cap and industry-by-time fixed effect. As a robustness check, we nevertheless take control variables suggested by other hypotheses on nominal price literature into consideration.[23] Table I summarize these variables as well as the ways to construct them.

The regression takes the following form:

$$Herfindahl\ Index_{\ i,t} = u_{j,t} + \beta \times tick_{relative_{i,t}} + \Gamma \times X_{i,t} + \epsilon_{i,t} \qquad (21)$$

where $Herfindahl\ Index_{\mathrm{i,t}}$ is the Herfindahl Index for stock i on date t. $u_{j,t}$ is the industry-by-time fixed effect. The key variable of interest, $tick_{relative_{i,t}}$, is the daily inverse of the stock price for stock i. $X_{i,t}$ are the control variables that include idiosyncratic risk, age, the number of analysts, small investor ownership, and the probability of informed trading.

Insert Table 2 about Here

Table 2 shows that market fragmentation increases with the relative tick size. For example, Column 1 shows that a one-standard-deviation increase in relative tick size increases Herfindahl Index by 0.015 (0.383*0.039), representing 4.7% （0.015/0.317） of the mean of the Herfindahl Index. Table 2 shows that trading of large firms is more fragmented, as is trading of firms with longer histories. Variables other than the relative tick size, market cap, and firm age do not have a significant impact on market fragmentation.

C. Splits/Reverse Splits as Exogenous Shocks to the Relative Tick Size

[23] The optimal tick size hypothesis argues that firms choose the optimal tick size through splits/reverse splits (Angel 1997; Anshuman and Kalay 2002). We include the idiosyncratic risk, and the number of analysts that may affect the choice of the optimal tick size, from this study. The marketability hypothesis argues that a lower price appeals to individual traders. We include the measure of small investor ownership suggested by Dyl and Elliott (2006), which is equal to the logarithm of the average book value of equity per shareholder. The signaling hypothesis states that firms use stock splits to signal good news. We use the probability of informed trading (PIN) proposed by Easley, Kiefer, O'Hara and Paperman (1996) to control for information asymmetry. Two lines of research do not suggest additional variables to control for in our study. The catering hypothesis by Baker, Greenwood, and Wurgler (2009) discusses time-series variations in stock prices: firms split when investors place higher valuations on low-priced firms and vice versa, but our analysis focuses on cross-sectional variation. Campbell, Hilscher, and Szilagyi (2008) find that an extremely low price predicts distress risk, but the 117 firms in our sample are far from default.

This section establishes the causal relationship between the relative tick size and market fragmentation following the identification strategy proposed by Yao and Ye (2015). The difference-in-differences test uses leveraged ETFs that split/reverse split as the pilot group, and leveraged ETFs that track the same index but do not split/reverse split as the control group.

Specifically, we estimate the following model:

$$Herfindahl\ Index_{\ i,j,t} = u_{i,t} + \gamma_{i,j} + \rho \times D_{trt_{i,t,j}} + \theta \times return_{i,t,j} + \epsilon_{i,t,j} \qquad (22)$$

where *i* indexes the underlying index, *j* indexes ETFs, and *t* indexes time. The dependent variable in the equation is the Herfindahl Index. We include index-by-time fixed effects, $u_{i,t}$, which controls for the time trend that may affect each index. $\gamma_{i,j}$ capture the ETF fixed effects that absorb the time-invariant differences between two leveraged ETFs that track the same index i. The regression also controls for their returns, $return_{i,t,j}$, in each period, which is the only main difference left between the ETFs tracking the same index after we control for index-by-time and ETF fixed effects. $D_{trt_{i,t,j}}$ is the treatment dummy, which equals 0 for the control group. For the treatment group, the treatment dummy equals 0 before splits/reverse splits and 1 after splits/reverse splits. The coefficient estimate ρ, for the dummy variable, captures the treatment effect.

To derive an unbiased estimate of the treatment effect, the actual splits/reverse splits must be uncorrelated with the error term. This does not mean that the actual split/reverse split must be exogenous. As we control for both index-by-time fixed effects and ETF fixed effects, the estimation will be biased only if the actual splits/reverse splits are somehow related to the contemporaneous idiosyncratic shocks to the dependent variables (Hendershott, Jones and Menkveld (2011)). Two stylized facts are then important for establishing the unbiasedness of the coefficient estimate.

First, the schedule for executing splits/reverse splits is predetermined and announced well before the actual splits/reverse splits.[24] Thus it seems highly unlikely that the splits/reverse splits schedule could be correlated with idiosyncratic shocks to HFT liquidity provision in the future. In

[24] Using a longer time window creates overlap between the pre-announcement and pre-split periods, but we find similar results.

addition, fund companies often conduct multiple splits/reverse splits on the same day for ETFs tracking diversified underlying assets.[25] Such a diversified sample further mitigates the concern that the splits/reverse splits decisions are correlated with ETF-specific idiosyncratic shocks. Second, the motivation for ETF splits/reverse splits is transparent. The issuers of ETFs conduct splits/reverse splits when their nominal prices differ dramatically from their pairs. Such differences in price can be captured by the ETF fixed effect, and the estimate of coefficient ρ remains unbiased.

Table 3 displays the regression result for the impacts of splits on the Herfindahl Index for Leveraged ETF splits and reverse splits. Column (1) indicates that the Herfindahl Index decreases by 0.047 after splits, implying that trading becomes more fragmented after an increase in the relative tick size. In terms of economic significance, a decrease of 0.047 represents a 16.7% decrease relative to the mean of Herfindahl Index of the leverage sample. Column (2) indicates that the Herfindahl Index increases by 0.014 after reverse splits, implying that trading becomes more consolidated after a decrease in the relative tick size.

Insert Table 3 about Here

VII. Conclusion

We examine the competition between stock exchanges over proposed make-take fees. When traders can quote a continuous price, the breakdown of the make-take fees is neutralized and order flow consolidates to the exchange with the lowest total fee. Under tick size constraints, fee breakdowns are no longer neutral, and such non-neutrality of fee structure explains a number of anomalies in price competition between stock exchanges. We first show that the two-sidedness of the market allows operators to establish multiple exchanges with heterogeneous fee structures for second-degree price discrimination. Second, we demonstrate the non-existence of pure-strategy

[25] For example, the announcement made on April 9, 2010 involves splits ranging from oil, gas, gold, real estate, financial stocks, and basic materials to Chinese indices.

equilibrium in the fee game under tick size constraints, which explains the diversity and frequent fluctuations in fee structures.

Price competition under the tick size constraints then explains the market fragmentation among nearly homogeneous stock exchanges. First, the same operator has an incentive to operate multiple exchanges to implement second-degree price discrimination. Second, mixed-strategy equilibria entail positive profits for all competing operators. This result explains the entry of platforms with new fee structures.

Our paper contributes to the literature on make-take fees and tick size, providing policy implications for the debate over the two topics. As make-take fees constitute transaction costs imposed on liquidity makers and liquidity takers, they have attracted regulators' attention. A regulation capping the take fee at 30 cents per one hundred shares has already been implemented, and more aggressive initiatives, such as banning the fee completely, are under discussion among regulators. One argument for banning the fees is based on fairness, because the fees lead to wealth transfer from one side of the market to the other. We show, however, that in the presence of tick size constraints, fees imposed by the exchanges can Pareto improve the market by proposing sub-tick prices. The second argument for banning the fee cites its complexity and frequent fluctuations, which can be explained by the mixed-strategy equilibria in the model. The last argument for banning the fee involves agency concerns. Recently, Battalio, Corwin, and Jennings (2014) find that broker/dealers have a strong incentive to route customers' limit orders to the market offering the highest rebate, because brokers/dealers are permitted to pocket such rebates. This conflict of interest leads to two policy proposals: (1) passing the rebate back to customers; (2) eliminating the fees (Angel, Harris, and Spatt (2010, 2013)). Our paper reveals an economic force that is in favor of the first solution. Passing the rebate back to customers is a direct solution to the agency issue, while eliminating the fees might hinder the would-be efficiency of trading within the tick size.

Our paper shows that make-take fees are market design response from competing exchanges to bypass the *existing* tick size regulations.[26] It questions the rationale of a recent

[26] For studies on the tick size, see Yao and Ye (2014 and 2015), O'Hara, Saar and Zhong (2014), and Buti et al. (2014), among others.

initiative to increase the tick size for small stocks to five cents. Encouraged by the JOBS Act, the SEC is proposing a pilot program to increase the tick size.[27] The motivation for increasing the tick size is that it may increase market-making profit and support sell-side equity research and, eventually, increase the number of IPOs (Weild, Kim and Newport (2012)). Our paper indicates, however, that exchanges can use fee structures to create cum fee prices within the tick. An increase in tick size can create more room for multiple platforms to co-exist and thus potentially a more fragmented market.

Our paper is subject to several limitations. For example, we do not model the competition between liquidity makers, we do not allow limit-order queuing, and we do not allow traders to choose their order types. It would be interesting to examine whether more richly structured models would lead to new insights into exchange competition. Also, liquidity makers and liquidity takers in our model directly execute their own orders, but the execution is often delegated to agents such as brokers or execution desks in current markets. It would be interesting to build agency issues into our model.

Our model is based on the exchange industry, but the intuition from the model can be applied to other two-sided markets. The central idea of the paper is that platform operators can exploit the inability to neutralize the fee structure from two sides, creating differentiation for otherwise homogeneous products. We believe one fruitful line of research would be to apply this intuition to other markets. For example, the non-surcharge provision in the credit card industry prevents merchants from charging differentiated prices for individual credit cards, and thus leads to non-neutrality of the fee structure. We conjecture that such non-neutrality might be a force leading to the proliferation of credit cards.

[27] "SEC Provides Details of 5-Cent Tick Test," *Wall Street Journal*, June 25, 2014.

Appendix A: Proofs of Propositions and Lemmas

Proof of Lemma 1:

We solve the model by backward induction. At Date 2, a liquidity taker sells if $v_s \leq P_i - f_t^i$. Thus, the probability that a seller accepts an order is $\max\{0, \frac{2}{d} \cdot (P_i - f_t^i)\}$, and the liquidity maker's expected profit for submitting a buy limit order at price P_i to platform i is $(v_b - P_i - f_m^i) \cdot \Pr(v_s \leq P_i - f_t^i)$. The choice of P_i reflects the following trade-off. A decrease in P_i increases the liquidity maker's gain conditional on execution $(v_b - P_i - f_m^i)$, but decreases the likelihood that the liquidity taker accepts her limit order $(\Pr(v_s \leq P_i - f_t^i)$. Therefore, the liquidity maker's maximal profit for choosing platform *i* is

$$\max_{P_i}(v_b - P_i - f_m^i) \cdot \Pr(v_s \leq P_i - f_t^i) = \max_{P_i}[(v_b - P_i - f_m^i) \cdot \max\left\{0, \frac{2}{d} \cdot (P_i - f_t^i)\right\}]$$

which yields

$$P_i^*(v_b) = \frac{v_b + T_i}{2} - f_m^i, \text{ if } v_b \geq T_i\,.$$ [28]

Therefore, fixing total fee T_i, an increase in the make fee f_m^i decreases the buy limit-order price proposed by the liquidity maker by the same amount, leading the cum fee buy price unchanged at $\frac{v_b+T_i}{2}$. Therefore, the change in the fee structure when holding total fee T_i fixed is neutralized by the liquidity maker. The neutrality of the fee structure can also be demonstrated by the liquidity taker's decision.

The maximal surplus for a buyer with valuation v_b who provides liquidity in platform *i* is:

$$BS_i(v_b) = (v_b - P_i^*(v_b) - f_m^i) \cdot \Pr(v_s \leq P_i^*(v_b) - f_t^i) = \frac{(v_b - T_i)^2}{2d},$$

as the liquidity maker proposes a limit order in platform *i* only when $v_b \geq T_i$, $BS_i(v_b)$ decreases in total fee T_i. The liquidity maker submits no limit order if the total fees of all platforms are higher

[28] If $v_b < T_i$, the maker cannot propose a limit order accepted by the taker without losing money.

than v_b; otherwise she submits a limit order at price $\frac{v_b+T_i}{2} - f_m^i$ to the platform with the lowest total fee.

A monopoly operator does not have incentives to open multiple platforms, because all liquidity makers choose the platform with the lowest total fee. As the duopoly operators have incentives to undercut each other's fee towards zero. ■

Proof of Lemma 2: If $f_m^i > 0$, the liquidity maker must propose a price at $P = 0$, because otherwise $P + f_m^i > d$. It follows that $f_t^i < 0$, because otherwise $P - f_t^i = -f_t^i < 0$. Similarly, if $f_m^i < 0$, liquidity maker must propose a price at $P = d$ and $f_t^i > 0$. Thus, a necessary condition for a trade to occur is to charge one side and subsidize the other side.

Meanwhile, for trades to occur we must have

$$\begin{cases} 0 \le P - f_t^i \\ P + f_m^i \le d \end{cases},$$

which is equivalent to

$$\begin{cases} f_t^i \le P \\ f_m^i \le d - P \end{cases}.$$

Thus,

$$f_m^i + f_t^i \le d.$$

The rest of the lemma follows directly. ■

Proof of Proposition 1: Under platform 1's fee structure (f_m, f_t), according to parts (ii) and (iii) of Lemma 2, the liquidity maker will propose a buy price at $P = 0$, and trade with the liquidity taker with $v_s \le \min\{-f_t, d/2\}$. So the liquidity maker's surplus when choosing platform 1 is

$$BS^1 = (v_b - f_m) \cdot \Pr(v_s \le \min\{-f_t, d/2\})$$
$$= \begin{cases} \frac{2}{d} \cdot (v_b - f_m) \cdot (-f_t) & if \ -f_t < d/2 \\ v_b - f_m & if \ -f_t \ge d/2 \end{cases}.$$

Under platform 2's fee structure (f_t, f_m), the liquidity maker will similarly propose a buy price at $P = d$, and trade with the liquidity taker with $v_s \le \min\{d - f_m, d/2\}$. So the liquidity maker's surplus when choosing platform 2 is

$$BS^2 = (v_b - d - f_t) \cdot \Pr(v_s \leq \min\{d - f_m, d/2\})$$

$$= \begin{cases} \frac{2}{d} \cdot (v_b - d - f_t) \cdot (d - f_m) & if\ f_m > d/2 \\ v_b - d - f_t & if\ f_m \leq d/2 \end{cases}.$$

We consider the following three possible cases:

Case (i): $f_m > -f_t \geq d/2$

$$BS^1 - BS^2 = v_b - f_m - \frac{2}{d} \cdot (v_b - d - f_t) \cdot (d - f_m)$$

$$= \frac{2}{d} \cdot \left[\left(f_m - \frac{d}{2} \right) \cdot v_b - \frac{d}{2} \cdot f_m + (d + f_t) \cdot (d - f_m) \right].$$

Note that $BS^1 - BS^2$ increases with v_b, because $f_m > d/2$. Hence,

$$BS^1 - BS^2 \leq [BS^1 - BS^2] | when\ v_b = d$$

$$= \frac{2}{d} \cdot (d - f_m) \cdot (\frac{d}{2} + f_t) \leq 0.$$

The liquidity maker thus prefers platform 2.

Case (ii): $f_m > \frac{d}{2} > -f_t$

$$BS^1 - BS^2 = \frac{2}{d} \cdot (v_b - f_m) \cdot (-f_t) - \frac{2}{d} \cdot (v_b - d - f_t) \cdot (d - f_m)$$

$$= \frac{2}{d} \cdot (f_m - f_t - d) \cdot (v_b - d).$$

So

$$BS^1 \gtreqless BS^2\ if\ and\ only\ if\ f_m - f_t \lesseqgtr d.$$

Case (iii): $d/2 \geq f_m > -f_t$

$$BS^1 - BS^2 = \frac{2}{d} \cdot (v_b - f_m) \cdot (-f_t) - v_b - d - f_t$$

$$= \frac{2}{d} \cdot \left[\left(-f_t - \frac{d}{2} \right) \cdot v_b - (-f_t) \cdot f_m + (d + f_t) \cdot \frac{d}{2} \right].$$

Note that $BS^1 - BS^2$ decreases with v_b, because $d/2 > -f_t$. Hence,

$$BS^1 - BS^2 \geq [BS^1 - BS^2] | when\ v_b = d$$

$$= \frac{2}{d} \cdot (-f_t) \cdot (\frac{d}{2} - f_m) \geq 0.$$

The liquidity maker thus prefers platform 1.

Combining the three cases above, the proposition follows. ■

Proof of Lemma 3: We consider the case when $p_s^1 < p_s^2$, as we do in the main text. That is, we focus on the rectangular area $(p_b^2, p_s^2) \in [\frac{d}{2}, d] \times [p_s^1, \frac{d}{2}]$. The analysis parallels the case in which $p_s^1 > p_s^2$.

When $p_b^1 \geq p_b^2$, we have $\varphi \leq p_b^2 \leq p_b^1$. So for any $v_b \geq p_b^2$, we have $v_b \geq \varphi$. Hence, $BS^2 - BS^1 \geq 0$ for any $v_b \geq p_b^2$.

When $p_b^1 < p_b^2$, we have $p_b^1 < p_b^2 < \varphi$. In this case, we can show that $\varphi > \frac{d}{2}$. Note that $\varphi = p_b^1 + (p_b^2 - p_b^1) \cdot \frac{p_s^2}{p_s^2 - p_s^1} > p_b^1 > \frac{d}{2}$, where the first inequality follows from $p_b^2 > p_b^1$ and $p_s^2 > p_s^1$. Thus, two possible cases are to be further considered: 1) If $\frac{d}{2} < \varphi < d$, then for any $p_b^1 \leq v_b \leq \varphi$, $BS^1 - BS^2 \geq 0$, and for any $\varphi < v_b \leq d$, $BS^1 - BS^2 < 0$. 2) If $d \leq \varphi$, then for any $v_b \leq d$, we have $v_b \leq \varphi$. So $BS^1 - BS^2 \geq 0$ for any $v_b \leq d$. ■

Proof of Proposition 2: it is a special case in the proof of Proposition 3 and the result follows the general formula in Proposition 3 with $k = 1$.

Proof of Proposition 3: Suppose a monopoly operator opens k platforms. For each platform i, p_b^i and p_s^i denote its cum fee buy and sell prices, where $1 \leq i \leq k$. Without loss of generality, we can assume $p_s^1 \leq p_s^2 \leq \cdots \leq p_s^k$. Based on the analysis from product differentiation, we must have $p_b^1 \leq p_b^2 \leq \cdots \leq p_b^k$. Otherwise, some platforms would have zero trading volume. For example, if platform i has a lower cum fee buy price and a higher cum fee sell price than platform j, a liquidity maker will strictly prefer platform i over platform j, and platform j will have zero trading volume. A monopoly operator would not open a platform with zero trading volume. Now we consider liquidity makers' segmentation under these k platforms. If $v_b < p_b^1$, the liquidity maker will post no limit order on any platform. If $p_b^1 \leq v_b < p_b^2$, the liquidity maker will post a limit order on platform 1. If $p_b^2 \leq v_b < p_b^3$, the liquidity maker will post a limit order on platform 1 only if:

$$\frac{2}{d} \cdot (v_b - p_b^2) \cdot p_s^2 \leq \frac{2}{d} \cdot (v_b - p_b^1) \cdot p_s^1,$$

which is equivalent to $v_b \leq \frac{p_b^2 p_s^2 - p_b^1 p_s^1}{p_s^2 - p_s^1}$. Denote $\varphi_1 = \frac{p_b^2 p_s^2 - p_b^1 p_s^1}{p_s^2 - p_s^1}$. A liquidity maker with valuation $v_b \epsilon [p_b^1, \varphi_1]$ will post a limit order on platform 1. Similarly, a liquidity maker with valuation $p_b^3 \leq v_b < p_b^4$ will post a limit order on platform 2 only if:

$$\frac{2}{d} \cdot (v_b - p_b^3) \cdot p_s^3 \leq \frac{2}{d} \cdot (v_b - p_b^2) \cdot p_s^2,$$

which is equivalent to $v_b \leq \frac{p_b^3 p_s^3 - p_b^2 p_s^2}{p_s^3 - p_s^2}$. Denote $\varphi_2 = \frac{p_b^3 p_s^3 - p_b^2 p_s^2}{p_s^3 - p_s^2}$. A liquidity maker with valuation $v_b \epsilon [\varphi_1, \varphi_2]$ will post a limit order on platform 2. We repeat this exercise for each of the k platforms. Denote

$$\varphi_i = \begin{cases} p_b^1 & \text{for } i = 0 \\ \dfrac{p_b^{i+1} p_s^{i+1} - p_b^i p_s^i}{p_s^{i+1} - p_s^i} & \text{for } 1 \leq i \leq k-1 \\ d & \text{for } i = k \end{cases}$$

A liquidity maker with valuation $v_b \epsilon [\varphi_{i-1}, \varphi_i]$ will post a limit order on platform i for $1 \leq i \leq k$. If there is no fixed cost involved in setting a new platform, the monopoly operator will choose its cum fee buy price p_b^i and cum fee sell price p_s^i on each platform i to maximize its total profit:

$$\pi^{ME}(k) = \frac{4}{d^2} \sum_{i=1}^{k} (\varphi_i - \varphi_{i-1}) \cdot p_s^i \cdot (p_b^i - p_s^i)$$

$$s.t. \varphi_0 \leq \varphi_1 \leq \cdots \leq \varphi_k$$

We first maximize the above profit function without the constraint on φ_i. The first order conditions are:

$$\frac{\partial \pi^{ME}(k)}{\partial p_b^i} = \frac{4}{d^2}[\frac{\partial \varphi_{i-1}}{\partial p_b^i} p_s^{i-1}(p_b^{i-1} - p_s^{i-1})$$

$$+ \left(\frac{\partial \varphi_i}{\partial p_b^i} - \frac{\partial \varphi_{i-1}}{\partial p_b^i}\right) p_s^i (p_b^i - p_s^i) + (\varphi_i - \varphi_{i-1}) p_s^i - \frac{\partial \varphi_i}{\partial p_b^i} p_s^{i+1}(p_b^{i+1} - p_s^{i+1})\Big] = 0,$$

$$\frac{\partial \pi^{ME}(k)}{\partial p_s^i} = \frac{4}{d^2}[\frac{\partial \varphi_{i-1}}{\partial p_s^i} p_s^{i-1}(p_b^{i-1} - p_s^{i-1}) + \left(\frac{\partial \varphi_i}{\partial p_s^i} - \frac{\partial \varphi_{i-1}}{\partial p_s^i}\right) p_s^i (p_b^i - p_s^i) + (\varphi_i - \varphi_{i-1})(p_b^i - 2p_s^i) -$$

$$\frac{\partial \varphi_i}{\partial p_s^i} p_s^{i+1}(p_b^{i+1} - p_s^{i+1})\Big] = 0, \; i = 1, 2, \cdots, k,$$

where we set $p_b^0 = p_s^0 = p_b^{k+1} = p_s^{k+1} = 0$; by solving the above $2k$ equations, we get the optimal cum fee buy and sell price for each platform i, where $1 \leq i \leq k$:

$$p_b^{i*} = \frac{d}{2} + \frac{i \cdot d}{2(2k+1)}, p_s^{i*} = \frac{i \cdot d}{2k+1}$$

Under these cum fee buy and sell prices, for $i = 0, 1, \cdots, k$ we have,

$$\varphi_i = \frac{k+i+1}{2k+1} d,$$

which satisfies the constraint that $\phi_0 \leq \phi_1 \leq \cdots \leq \phi_k$. So the above cum fee buy and sell prices are also solutions to the constraint maximization problem. By establishing k platforms, the profit of the monopoly operator is:

$$\pi^M(k) = \frac{4}{d^2} \sum_{i=1}^{k} \left(\frac{\mathrm{k+i+1}}{\mathrm{2k+1}} d - \frac{k+i}{2k+1} d\right) \cdot \frac{i \cdot d}{2k+1} \cdot \left(\frac{d}{2} + \frac{i \cdot d}{2(2k+1)} - \frac{i \cdot d}{2k+1}\right)$$

$$= \frac{2k \cdot (k+1) \cdot d}{3(2k+1)^2}$$

If a fixed cost c is involved in opening a new platform, the monopoly operator will open the k_{th} platform only if:

$$\pi^M(k) - \pi^M(k-1) = \frac{4k \cdot d}{3(2k+1)^2(2k-1)^2} \geq c.$$

The left-hand side is the profit increment by opening the k^{th} platform, while the right-hand side is the cost for opening an additional platform. The monopoly operator will continue to open platforms as long as the profit increment is larger than the fixed cost c of opening a new platform.

The maker's surplus is

$$BS^M(k) = \frac{4}{d^2}\sum_{i=1}^{k}(\varphi_i - \varphi_{i-1}) \cdot p_s^i \cdot \left(E\left(v_b \middle| \varphi_{i-1} \leq v_b \leq \varphi_i\right) - p_b^i\right)$$
$$= \frac{4}{d^2}\sum_{i=1}^{k}\left(\frac{\mathrm{k+i+1}}{\mathrm{2k+1}}d - \frac{k+i}{2k+1}d\right) \cdot \frac{i \cdot d}{2k+1}$$
$$\cdot \left[\frac{1}{2}\left(\frac{k+i+1}{2k+1}d + \frac{k+i}{2k+1}d\right) - \frac{d}{2} - \frac{i \cdot d}{2(2k+1)}\right] = \frac{k \cdot (k+1)}{3(2k+1)^2} \cdot d$$

The liquidity taker's suplus is

$$SS^M(k) = \frac{4}{d^2}\sum_{i=1}^{k}(\varphi_i - \varphi_{i-1}) \cdot p_s^i \cdot (p_s^i - E\left(v_s \middle| v_s \leq p_s^i\right))$$
$$= \frac{4}{d^2}\sum_{i=1}^{k}\left(\frac{\mathrm{k+i+1}}{\mathrm{2k+1}}d - \frac{k+i}{2k+1}d\right) \cdot \frac{i \cdot d}{2k+1} \cdot \left(\frac{i \cdot d}{2k+1} - \frac{1}{2}\frac{i \cdot d}{2k+1}\right)$$
$$= \frac{k \cdot (k+1)}{3(2k+1)^2} \cdot d$$

This proves the proposition. ■

Proof of Proposition 4 (By contradiction) Without loss of generality, suppose a pure-strategy equilibrium exists, and in equilibrium $\pi^1 \geq \pi^2$. Two possible cases are to be considered: (i) $\pi^1 > 0$; (ii) $\pi^1 = \pi^2 = 0$.

(i) There are two subcases: (i-a) $\pi^1 > \pi^2 \geq 0$; (i-b) $\pi^1 = \pi^2 > 0$.

(i-a) Platform 2 can set its fees such that

$$p_b^2 = p_b^1 - \varepsilon \ and \ p_s^2 = p_s^1, \tag{A.1}$$

where $\varepsilon > 0$. The above conditions correspond to area s_6. By Lemma 5, liquidity makers no longer go on platform 1, and operator 2's profit becomes

$$\hat{\pi}^2 = (p_b^1 - p_s^1 - \varepsilon) \cdot \ \Pr(v_b \geq \hat{v}_b^2) \cdot \Pr(v_s \leq \hat{v}_s^2)$$

$$\geq (p_b^1 - p_s^1 - \varepsilon) \cdot \Pr(v_b \geq \hat{v}_b^1) \cdot \Pr(v_s \leq \hat{v}_s^1)$$
$$= \pi^1 - \varepsilon \cdot \Pr(v_b \geq \hat{v}_b^1) \cdot \Pr(v_s \leq \hat{v}_s^1),$$

where the inequality follows from (A.1). Clearly, as long as $\pi^1 > \pi^2$, operator 2 can always strictly increase its profit by deviation (A.1) with a sufficiently small ε.

(i-b) If both platforms earn identical profits, platform 2 can set its fees as in A.1 to attract the entire market share. As long as ε is small enough, platform 2's profit will be sufficiently close to double its original profit. This definitely increases platform 2's profit. This shows that two platforms having the same positive profits cannot be a Nash-equilibrium.

(ii) There are two subcases: (ii-a) No trading; (ii-b) trading with $p_b^i = p_s^i \in (0, d), i = 1,2$.

(ii-a) No trading implies that $p_b^i \geq d$ or $p_s^i \leq 0$ for $i = 1,2$. Then platform 2 can set its fees such that $0 < p_s^2 < p_b^2 < d$. So the liquidity maker with valuation $v_b \geq \hat{v}_b^2$ will trade with the liquidity taker with valuation $v_s \leq \hat{v}_s^2$, and $\hat{\pi}^2 > 0$.

(ii-b) Denote $p_b^1 = p_s^1 = a$. Three subsubcases are to be further considered: (ii-b-I): $0 < a < \frac{d}{2}$; (ii-b-II): $a = \frac{d}{2}$; (ii-b-III): $\frac{d}{2} < a < d$.

(ii-b-I): Platform 2 can set its fees such that

$$p_b^2 = a + \varepsilon \text{ and } p_s^2 = a + \mu \cdot \varepsilon, \tag{A.2}$$

where $\varepsilon > 0$ and $0 < \mu < 1$. For sufficiently small ε, we have $\hat{v}_s^2 = p_s^2 > a = \hat{v}_s^1, p_b^2 > p_b^1$, and

$$\hat{\varphi} = p_b^2 + (p_b^2 - p_b^1) \cdot \frac{\hat{v}_s^1}{\hat{v}_s^2 - \hat{v}_s^1}.$$
$$= a\left(1 + \frac{1}{\mu}\right) + \varepsilon.$$

Clearly, $\hat{\varphi}$ decreases with μ , and $\lim_{\mu \to 1} \hat{\varphi} = 2a + \varepsilon < d$, where the inequality follows from $a < \frac{d}{2}$ given sufficiently small ε. Hence, for sufficiently small ε, we always have $\hat{\varphi} < d$. And from the liquidity makers' segmentation analysis, we know that the liquidity makers with $v_b \geq \hat{\varphi}$ will go on platform 2. So liquidity makers with high valuations will go on platform 2 and operator 2 will earn a strictly positive profit, as $p_b^2 - p_s^2 = (1 - \mu) \cdot \varepsilon$.

(ii-b-II): Platform 2 can set its fees such that

$$p_b^2 = a - \mu \cdot \varepsilon \; and \; p_s^2 = a - \varepsilon, \tag{A.3}$$

where $\varepsilon > 0$ and $0 < \mu < 1$ For sufficiently small ε, we have $\hat{v}_s^2 = p_s^2 < a = \hat{v}_s^1, p_b^2 < p_b^1$, and

$$\hat{\varphi} = p_b^1 + (p_b^1 - p_b^2) \cdot \frac{\hat{v}_s^2}{\hat{v}_s^1 - \hat{v}_s^2}.$$

$$= a(1 + \mu) - \mu \cdot \varepsilon.$$

Clearly, $\hat{\varphi}$ increases with μ, and $\lim_{\mu \to 1} \hat{\varphi} = d - \varepsilon < d$ and $\lim_{\mu \to 1} \hat{\varphi} = d - \varepsilon > \frac{d}{2}$ where the inequality follows from $a = \frac{d}{2}$ and $\varepsilon > 0$. Hence, for sufficiently small ε, we always have $\frac{d}{2} < \hat{\varphi} < d$. And from the liquidity makers' segmentation analysis, we know that liquidity makers with $\frac{d}{2} \leq v_b \leq \hat{\varphi}$ will go on platform 2. So liquidity makers with low valuations will go on platform 2 and operator 2 will earn strictly positive profit, as $p_b^2 - p_s^2 = (1 - \mu) \cdot \varepsilon$.

(ii-b-III): Platform 2 can set its fees such that

$$p_b^2 = a - \varepsilon \; and \; p_s^2 = \frac{d}{2}, \tag{A.4}$$

where $\varepsilon > 0$ and $a - \varepsilon > \frac{d}{2}$. We have $\hat{v}_s^2 = \frac{d}{2} = \hat{v}_s^1$, and $p_b^2 < p_b^1$. Then the liquidity maker with $v_b \geq p_b^2$ will trade with the liquidity taker with $v_s \leq \hat{v}_s^2$, and $\hat{\pi}^2 > 0$, as $p_b^2 - p_s^2 = a - \varepsilon - \frac{d}{2} > 0$. ■

Proof of Proposition 5:

(i) We establish *(i)* in Proposition 5 in the following 4 steps.

Step 1: $0 \leq p_s \leq p_b \leq d$.

Suppose that $p_b > d$ or $p_s < 0$ occurs with some positive probability in equilibrium. Note that these cases result in zero profits for the operator. One operator can always deviate by shifting such a probability to a strategy with $0 \leq \hat{v}_s < \hat{v}_b \leq d$, so that it will earn strictly positive profit with that probability.

Step 2: No mass point in the mixed-strategy equilibrium strategy.

There are two possible mass points to be considered: (a) some (p_b, p_s) with $p_b > p_s$; (b) some (p_b, p_s) with $p_b = p_s$. In case (a), a profitable deviation is given by (A.1). In case (b), a profitable deviation is given by (A.2), (A.3), and (A.4), respectively, for $0 < p_b = p_s < \frac{d}{2}, p_b = p_s = \frac{d}{2}$ and $\frac{d}{2} < p_b = p_s < d$.

Step 3: (p_b, p_s) *has a convex support on* $\left[\frac{d}{2}, d\right] \times \left[0, \frac{d}{2}\right]$.

First, given $p_b^i \geq \frac{d}{2}$, any $p_b^j < \frac{d}{2}$ is strictly dominated by $p_b^j = \frac{d}{2}$ for operator j, because a cum fee buy price lower than $\frac{d}{2}$ cannot attract more liquidity makers, and it can only lower the platform's per-unit profit. Similarly, we can rule out the $p_s > \frac{d}{2}$ strategy.

Second, the support of the mixed strategy must be convex. Suppose there is an unconnected support $[\alpha, \beta]$ and$[\gamma, \delta]$. By symmetry, the other operator would not randomize over the "hole" interval $[\beta, \gamma]$. However, in that case one operator will not be indifferent between choosing β and γ, which is a necessary condition for the operator to randomize over these two intervals. Thus, the support must be convex.

Step 4: There exists symmetric mixed-strategy equilibrium, and both operators earn strictly positive profits.

Given our pervious 3 steps, the existence of symmetric mixed-strategy equilibrium can be established by applying Theorem 6 in Dasgupta and Maskin (1986). The support ranges for p_b^i and p_s^i in Step 3 imply that both operators earn strictly positive profits in equilibrium.

(ii) Given $(p_b^1, p_s^1; p_b^2, p_s^2)$ in the possible support of the symmetric mixed-strategy equilibria

$$R \equiv \left[\frac{d}{2}, d\right] \times \left[0, \frac{d}{2}\right],$$

the liquidity makers' segmentation is given by Lemma 5, as shown in Figure 3.

If we standardize all terms above by dividing them by d, e.g., $P_b^i = \frac{p_b^i}{d}$ $(i = 1,2)$, then we establish a one-to-one mapping between region R and a half-unit square $\tilde{R} \equiv [\frac{1}{2}, 1] \times [0, \frac{1}{2}]$. Define $P(P_b^2) \equiv \frac{P_s^1 \cdot (1-P_b^1)}{1-P_b^2}, \Omega(P_s^2) \equiv 1 - \frac{P_s^1 \cdot (1-P_b^1)}{1-P_s^2}$, which corresponds to $\rho(p_b^2)$ and $\omega(p_s^2)$ in non-standardized variables, respectively. Areas s_1 to s_6, after standardization, correspond to the liquidity makers' segmentation in $\tilde{R}$ is shown in Figure A-1.

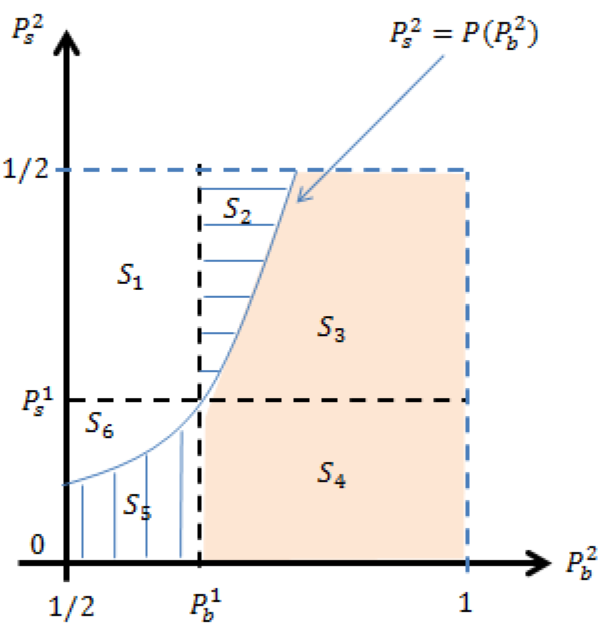

Figure A-1. Liquidity makers' segmentation under two operators after standardization. This figure shows liquidity makers' segmentation given $\left(P_b^1, P_s^1\right) \in \left[\frac{1}{2}, 1\right] \times \left[0, \frac{1}{2}\right]$, when $\left(P_b^2, P_s^2\right)$ can vary in the square $\left[\frac{1}{2}, 1\right] \times \left[0, \frac{1}{2}\right]$. The areas in this figure correspond to those in Lemma 5. In area S_1 and S_6, no liquidity maker chooses platform 1, and all liquidity makers choose platform 2; in area S_2, liquidity makers with low valuations choose platform 1, while liquidity makers with high valuations choose platform 2; in area S_3 and S_4, all liquidity makers choose platform 1, and no liquidity makers choose platform 2; in area S_5, liquidity makers with high valuations choose platform 1, while liquidity makers with low valuations choose platform 2.

$$S_1 \equiv \left\{(P_b^2, P_s^2) \left| \frac{1}{2} \le P_b^2 \le P_b^1, P_s^1 \le P_s^2 \le \frac{1}{2}\right.\right\},$$

$$S_2 \equiv \left\{(P_b^2, P_s^2) | P_b^1 \le P_b^2 \le \Omega(P_s^2), P_s^1 \le P_s^2 \le \frac{1}{2}\right\},$$

$$S_3 \equiv \left\{(P_b^2, P_s^2) | P_b^1 \le P_b^2 \le 1, P_s^1 \le P_s^2 \le \min\left\{P(P_b^2), \frac{1}{2}\right\}\right\},$$

$$S_4 \equiv \{(P_b^2, P_s^2) | P_b^1 \le P_b^2 \le 1, 0 \le P_s^2 \le P_s^1\},$$

$$S_5 \equiv \left\{(P_b^2, P_s^2) \left| \frac{1}{2} \leq P_b^2 \leq P_b^1, 0 \leq P_s^2 \leq P(P_b^2) \right.\right\},$$

$$S_6 \equiv \left\{(P_b^2, P_s^2) \left| \frac{1}{2} \leq P_b^2 \leq P_b^1, P(P_b^2) \leq P_s^2 \leq P_s^1 \right.\right\}$$

Recall that, with standardization,

$$V_j = \frac{v_j}{d}, (j = b, s).$$

Now $V_b \sim U\left[\frac{1}{2},\ 1\right]$, $V_s \sim U\left[0, \frac{1}{2}\right]$, and

$$\Phi \equiv \frac{\varphi}{d} = \frac{P_b^1 \cdot P_s^1 - P_b^2 \cdot P_s^2}{P_s^1 - P_s^2}.$$

Denote the support of the symmetric mixed-strategy equilibrium in $\tilde{R}$ as M.

When operator 2 plays a mixed strategy (P_b^2, P_s^2) with a distribution function $F(x, y)$ over M, operator 1's expected profit when choosing (P_b^1, P_s^1) is

$$\pi(p_b^1, p_s^1) = (p_b^1 - p_s^1) \cdot \Pr(v_s \leq p_s^1) \ \cdot [\iint_{S_2 \cap M} \Pr(P_b^1 \leq V_b \leq \Phi)\, dF(P_b^2, P_s^2)$$

$$+ \iint_{(S_3 \cup S_4) \cap M} \Pr(P_b^1 \leq V_b \leq 1)\, dF(P_b^2, P_s^2) + \iint_{S_5 \cap M} \Pr(\Phi \leq V_b \leq 1)\, dF(P_b^2, P_s^2)]$$

$$= 4 \cdot \mathrm{d} \cdot \Pi(P_b^2, P_s^2),$$

where $\Pi(P_b^2, P_s^2) \equiv (P_b^1 - P_s^1) \cdot\ P_s^1 \cdot \left[\iint_{S_2 \cap M} (\Phi - P_b^1)\, dF(P_b^2, P_s^2) + \iint_{(S_3 \cup S_4) \cap M} (1 - P_b^1)\, dF(P_b^2, P_s^2) + \iint_{S_5 \cap M} (1 - \Phi)\, dF(P_b^2, P_s^2)\right].$

Clearly, $\Pi(P_b^1, P_s^1)$ is independent of d, and part (ii) follows.

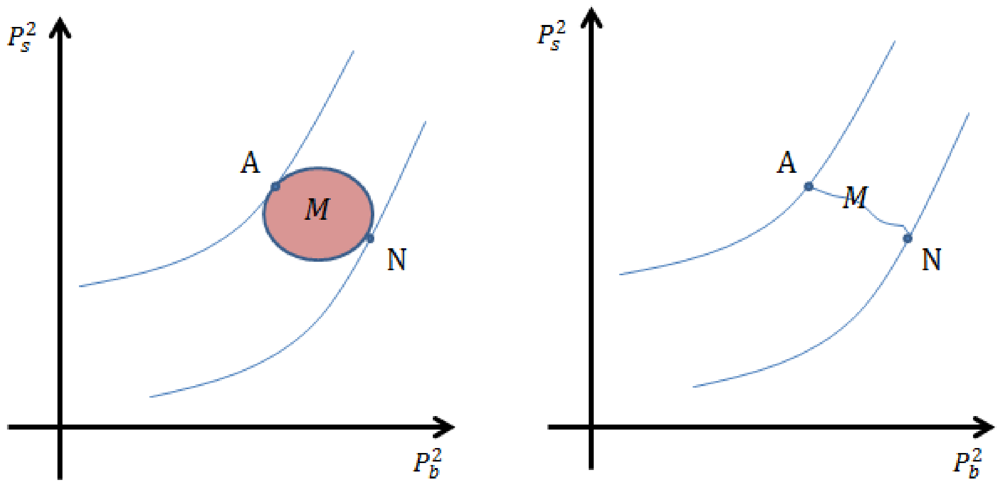

Figure A-2. The support of the mixed-strategy equilibrium cannot be either (a) a two-dimensional area, or (b) any downward-sloping curves. Panel (a) shows that for any two-dimensional area, we always can find two points A and N such that, at point A all liquidity makers choose platform 1, while at point N no liquidity maker chooses platform 1, so M cannot be a support of the mixed-strategy equilibrium. Panel (b) shows a similar logic for any downward-sloping curve.

(iii) From (ii), we focus without the loss of generality on $\Pi(P_b^2, P_s^2)$ and $\tilde{R}$.

First, we show that the support of the symmetric mixed-strategy equilibria cannot be a two-dimensional area.

(By contradiction) Suppose M is an area. From (i), we know that M must be convex.

However, for any possible convex area, we can always find two points A and N such that,

- at A, the curve $P_s^2 = P(P_b^2)$ is tangent and above M. So according to Figure A-1, all liquidity makers choose platform 1, and thus $\Pi > 0$ at A.
- at N, the curve $P_s^2 = P(P_b^2)$ is tangent and below M. So, according to Figure A-1, no liquidity makers choose platform 1, and thus $\Pi = 0$ at A.

So it is impossible that the profits at these two points are the same. Thus, M cannot be an area, as illustrated in panel (a) of Figure A-2. In other words, M must be a one-dimensional curve.

By the same argument, we can show that curve M cannot have any downward-sloping part, as illustrated in panel (b) of Figure A-2. Hence, part (iii) follows. ■

Proof of Corollary 1: For simplicity of notation, we here standardize all variables by d as we do in the proof of Proposition 5 (so $p_b = x \cdot d$), and denote the standardized cum fee buy price and cum free sell price on platform 1 as (a, b), and those established by platform 2's as (x, y). We take a guess and verify the approach. We guess that $y = x - \frac{1}{6}$ is indeed in equilibrium with distribution $F(x)$ on $[L, U]$, where $[L, U] \equiv \left[\frac{1}{2}, \frac{7}{12}\right]$.

The borderline $y = P(x) = \frac{(1-a)\cdot b}{1-x} = \frac{(1-a)(a-\frac{1}{6})}{1-x}$. $P'(a) = \frac{a-\frac{1}{6}}{1-a} \gtreqless 1$, if $a \gtreqless \frac{7}{12}$. Thus, for $a \in [L, U] = \left[\frac{1}{2}, \frac{7}{12}\right]$, the crossing of $y = P(x)$ is shown in Figure A-3 below.

Note that $\Phi(x, y) = \frac{a \cdot b - x \cdot y}{b - y} = a + x - \frac{1}{6}$. Thus, the profit of the platform established by operator 1 is

$$\Pi(a) = (a - b) \cdot b \cdot \left[\int_a^U \left(x - \frac{1}{6}\right) \cdot dF(x) + \int_L^a \left(1 - a - x + \frac{1}{6}\right) \cdot dF(x)\right]$$

$$= \frac{1}{6} \cdot \left(a - \frac{1}{6}\right) \cdot \left[\int_a^U x \cdot dF(x) - \int_L^a x \cdot dF(x) + \left(1 - a + \frac{1}{3}\right) \cdot F(a) - \frac{1}{6}\right]$$

Clearly, only S_2 and S_5 are possible.

$$\Pi'(a) = \frac{1}{6} \cdot [\int_a^U x \cdot dF(x) - \int_L^a x \cdot dF(x) + \left(\frac{3}{2} - 2a\right) \cdot F(a) - \frac{1}{6} + (a - \frac{1}{6})(\frac{4}{3} - 3a) \cdot f(a)]$$

$$\Pi''(a) = \frac{1}{6} \cdot [\left(a - \frac{1}{6}\right) \cdot \left(\frac{4}{3} - 3a\right) \cdot F''(a) + 10 \cdot \left(\frac{1}{3} - a\right) \cdot F'(a) - 2F(a)].$$

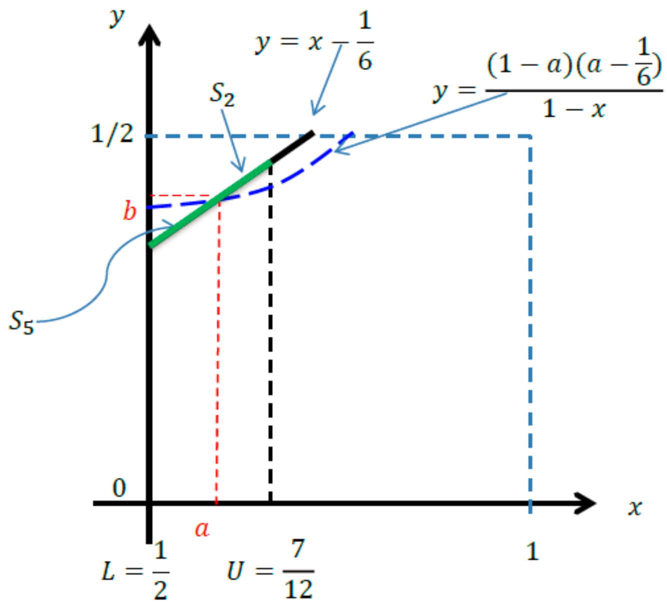

Figure A-3. One set of mixed-strategy equilibria. This figure shows the support of one set of mixed-strategy equilibria $[L, U]$ (in Green) on a straight line $y = x - \frac{1}{6}$.

strategy equilibria requires $\Pi'(a) = 0$ for any $a \in [L, U]$, which implies

$$\Pi''(a) = 0 \; for \; any \; a \in [L, U].$$

Solving this equation yields the parameterized distribution function in Equation (25).

To pin down the equilibrium, we need 2 parameters (C_1, C_2) that satisfy (26) and (27), where (26) is essentially $\Pi'(a) = 0$ for any $a \in [L, U]$. Equation (27) is required so that for $F(x)$ can be a legitimate distribution function. Because we have 2 parameters and essentially 1 equation to be satisfied, there are a set of parameters, and thus a set of equilibria. ■

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Figure 1. Structure of U.S. Stock Exchanges (Platforms) and Fee Structures in May 2015
This figure displays the ten U.S. stock exchanges (platforms) run by three holding companies (operators). The figure shows the make-take fee on each platform for NYSE-listed stocks in May 2015. For platforms offering more than one pricing tier, the standard rates are presented in the figure.

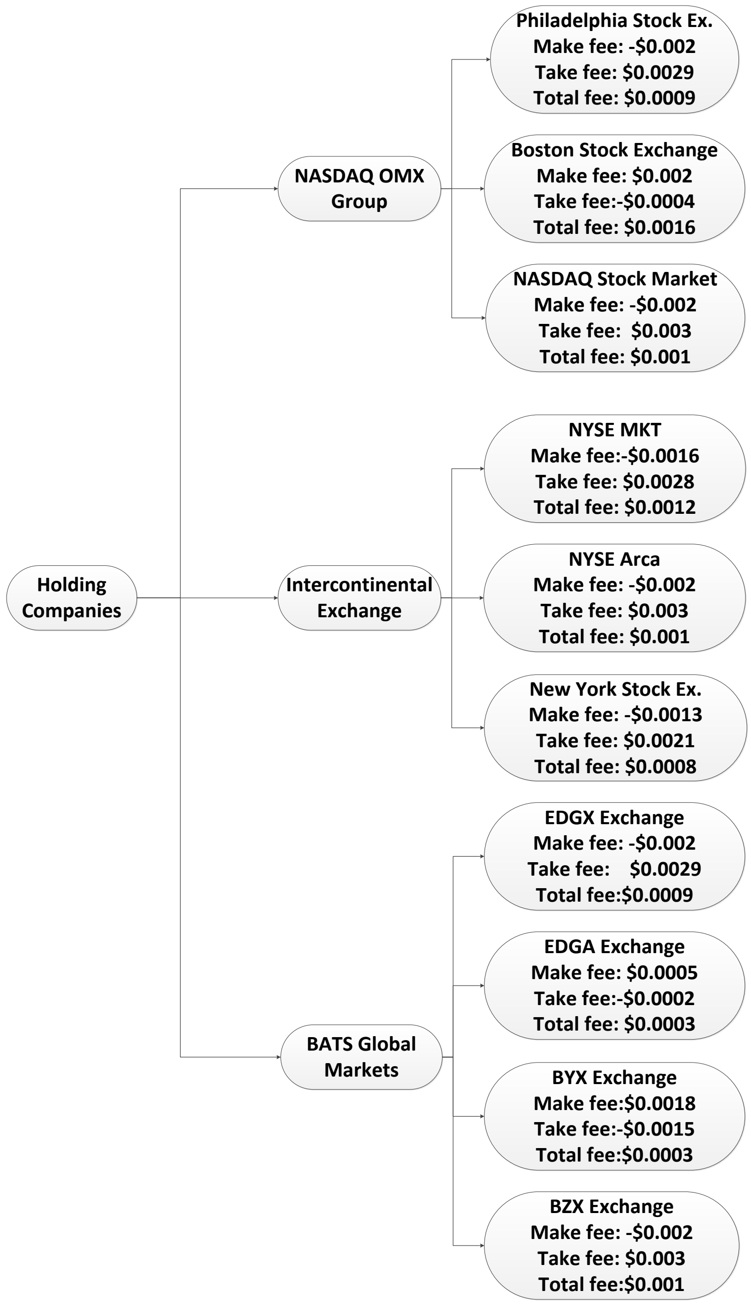


Figure 2. Liquidity Makers' Surpluses under Two Platforms. This figure shows the liquidity maker's surplus when choosing platforms. Without loss of generality, we assume that $p_s^1 < p_s^2$. The liquidity maker will choose the platform that offers her a higher surplus. Thus, the liquidity maker's choice is depicted as the upper envelope (in red) of the two surplus curves. Note that liquidity makers' valuations have an upper bound d.

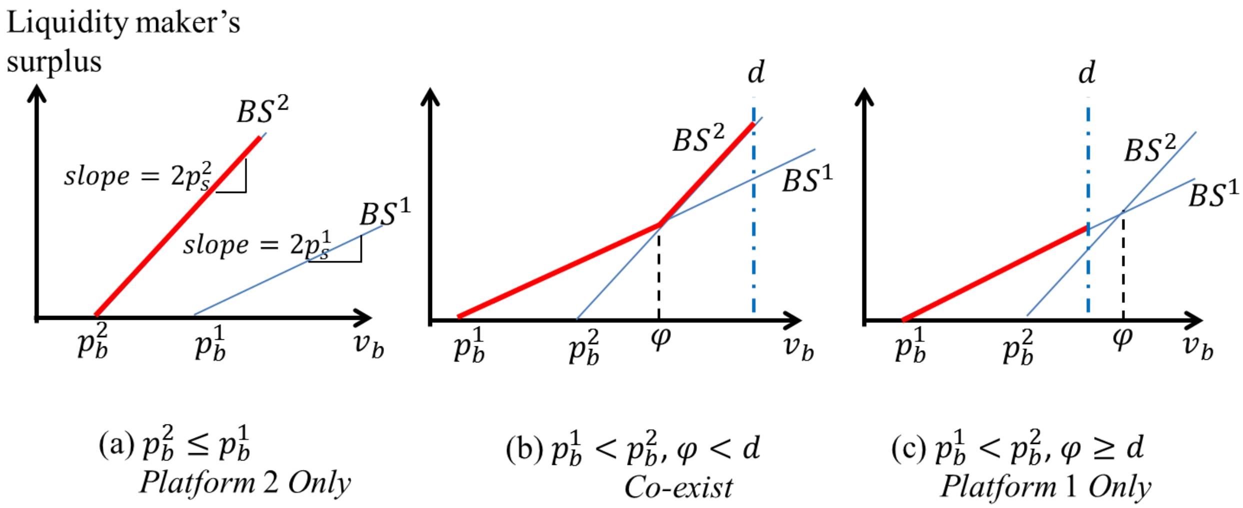


Figure 3. Liquidity Makers' Segmentation under Two Platforms. This figure shows the liquidity makers' segmentation given $(p_b^1, p_s^1) \in \left[\frac{d}{2}, d\right] \times \left[0, \frac{d}{2}\right]$, when (p_b^2, p_s^2) can vary in the square $\left[\frac{d}{2}, d\right] \times \left[0, \frac{d}{2}\right]$. As demonstrated in Lemma 4, in areas_1 and s_6, no liquidity maker chooses platform 1, and all liquidity makers choose platform 2; in area s_2, liquidity makers with low gains from trade choose platform 1, and liquidity makers with high gains from trade choose platform 2; in area s_3 *and* s_4, all liquidity makers choose platform 1, and no liquidity maker chooses platform 2; in area s_5, liquidity makers with high gains from trade choose platform 1, and liquidity makers with low gains from trade choose platform 2.

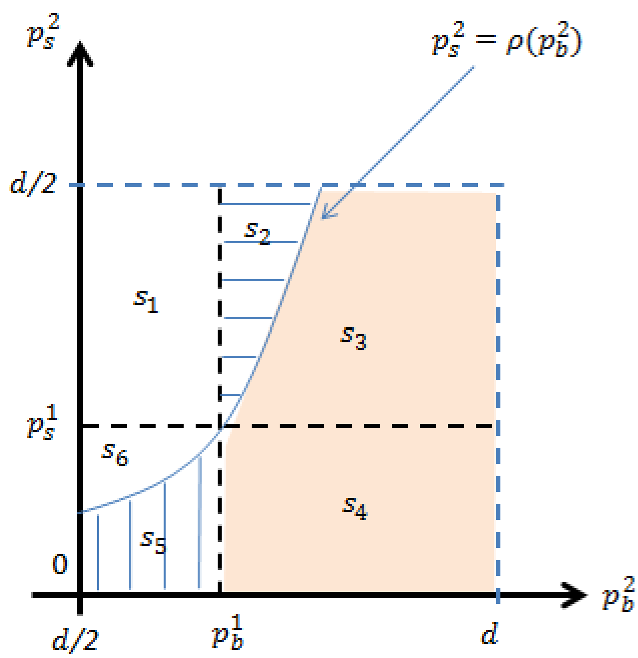


Figure 4. Liquidity Makers' Segmentation under Price Discrimination when One Monopoly Operator Runs Two Platforms. This figure shows the optimal cum fee buy and sell prices offered by two platforms run by one monopoly operator, and the corresponding liquidity makers' segmentation.

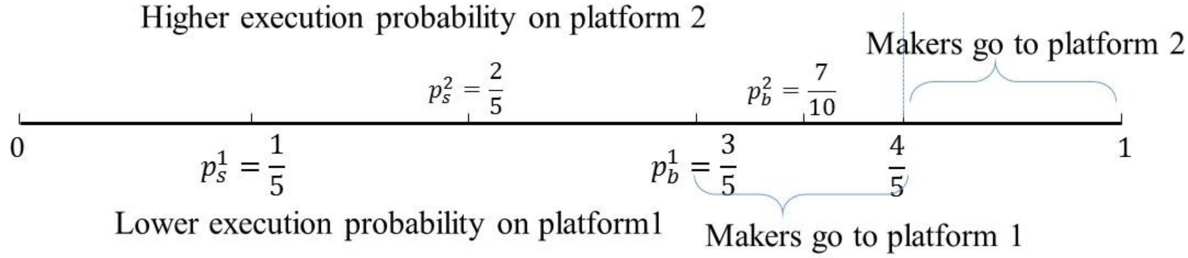


Figure 5. Two Types of Deviations from Bertrand Equilibrium. This figure demonstrates two types of profitable deviations for platform 2 when both platform 1 and 2 start by setting a zero total fee. Panel (a) demonstrates the deviation when platform 1 sets $p_b^1 = p_s^1 = \frac{1}{2} \cdot d$. The deviation of platform involves decreasing the cum fee buy price to $p_b^2 = \frac{1}{2}d - \mu\varepsilon$ and the cum fee sell price to $p_s^2 = \frac{1}{2}d - \varepsilon$, which attracts liquidity makers with low valuations. Panel (b) demonstrates the deviation when platform 1 sets $p_b^1 = p_s^1 < \frac{1}{2} \cdot d$. The deviation of platform 2 involves increasing the cum fee buy price to $p_b^2 = p_b^1 + \varepsilon$ and the cum fee sell price to $p_s^2 = p_s^1 + \mu \cdot \varepsilon$, which attracts liquidity makers with high valuations. $\varepsilon > 0$ and $0 < \mu < 1$ in both panels.

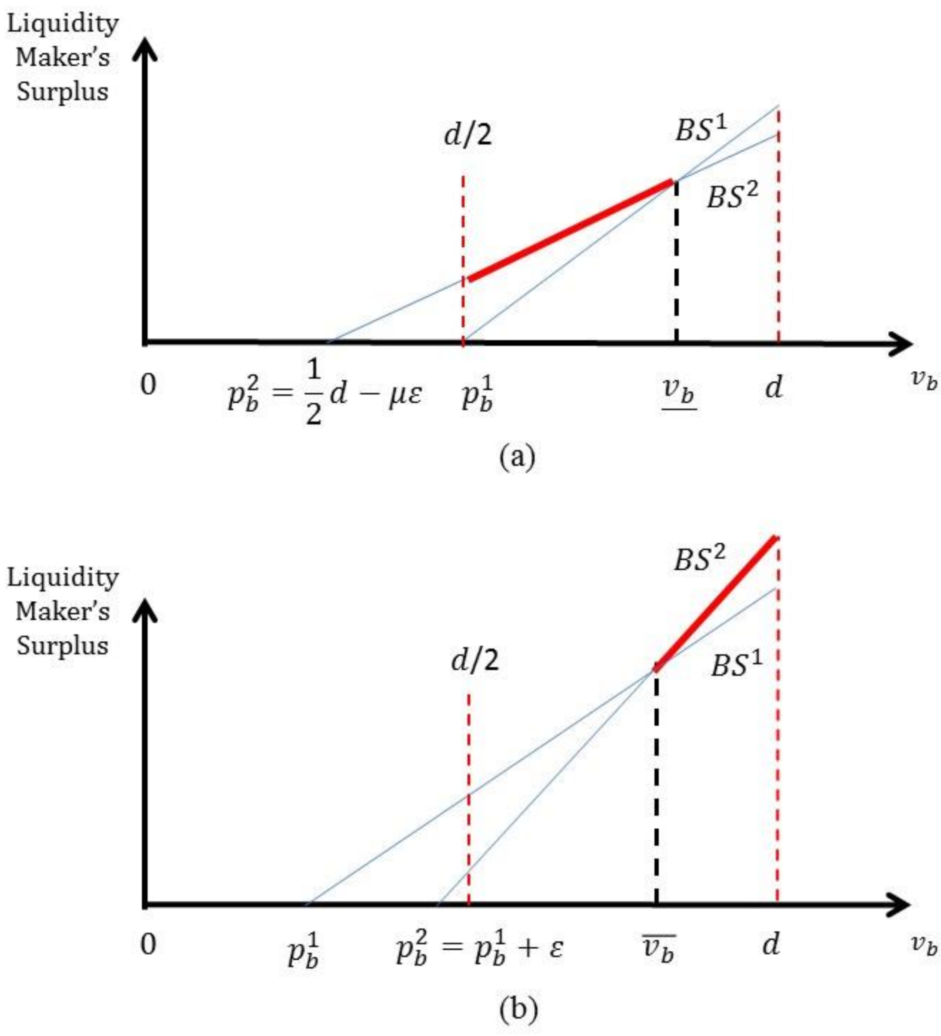


Table 1. Summary Statistics

This table reports summary statistics for the two samples used in the empirical tests. Panel A presents the summary statistics for the same 117-stock sample as the NASDAQ HFT dataset from January 2010 through November 2011. Panel B provides the summary statistics for the leveraged ETF sample used in the difference-in-differences test, in which the split/reverse split event happens between January 2010 and November 2011. *Herfindahl Index* is the Herfindahl Index of the security, used as a measure of market fragmentation. $tick_{relative}$ is the reciprocal of price. *logmcap* stands for the log value of market capitalization. $logbv_{average}$ is the logarithm of the average book value of equity per shareholder at the end of the previous year (December 2009). *idiorisk* is a measure of idiosyncratic risk pertaining to the security, calculated as the variance on the residual from a 60-month beta regression using the CRSP Value Weighted Index. *age* (in 1k days) is the length of time for which price information is available for a firm on the CRSP monthly file. *numAnalyst* is the number of analysts providing one-year earnings forecasts calculated from I/B/E/S. *PIN* is the probability of informed trading. *Return* stands for the daily return on the security.

	Mean	Median	Std.	Min.	Max.
Panel A. NASDAQ HFT Sample					
Herfindahl Index	0.317	0.304	0.089	0.123	0.921
$tick_{relative}$	0.048	0.033	0.039	0.002	0.364
logmcap	22.02	21.45	1.90	18.69	26.70
$logbv_{average}$	13.19	13.22	2.15	4.94	18.11
idiorisk	0.014	0.008	0.020	0.001	0.149
age (in 1k days)	9.77	7.56	7.86	0.68	31.38
numAnalyst	14.08	11.00	10.51	1.00	54.00
PIN	0.118	0.112	0.052	0.021	0.275
Panel B. Leveraged ETF Sample					
Herfindahl Index	0.281	0.270	0.076	0.163	0.844
return	-0.000	-0.001	0.041	-0.245	0.249

Table 2. The Impact of the Relative Tick Size on Market Fragmentation

This table presents the results of the regression of market fragmentation on the relative tick size. The regression uses the same 117-stock sample as the NASDAQ HFT dataset from January 2010 through November 2011. The regression specification is:

$$Herfindahl\ Index_{i,t} = u_{j,t} + \beta \times tick_{relative_{i,t}} + \Gamma \times X_{i,t} + \epsilon_{i,t}$$

where $MktFragmentation_{i,t}$ is measured using Herfindahl Index for stock *i* on date *t*. $tick_{relative_{i,t}}$ is the inverse of the stock price for stock *i* on day *t*. $u_{j,t}$ represents industry-by-time fixed effects. The definitions of the control variables $X_{i,t}$ are presented in Table 1. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Dep. Variable	*Herfindahl Index*				
	(1)	(2)	(3)	(4)	(5)
$tick_{relative}$	-0.383***	-0.385***	-0.312**	-0.362***	-0.289**
	(0.11)	(0.11)	(0.12)	(0.11)	(0.11)
logmcap	-0.023***	-0.023***	-0.020***	-0.021***	-0.017***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$logbv_{average}$		-0.000			-0.003
		(0.00)			(0.00)
idiorisk			-0.210		-0.232
			(0.18)		(0.17)
age			-0.002***		-0.002***
			(0.00)		(0.00)
numAnalyst			0.000		0.000
			(0.00)		(0.00)
PIN				0.139	0.145
				(0.10)	(0.09)
R^2	0.250	0.250	0.264	0.253	0.270
N	51950	51950	51950	51950	51950

Table 3. Difference-in-differences Test Using Leveraged ETF Splits (Reverse Splits)

This table presents the results of difference-in-differences tests using leveraged ETF split (and reverse split), in which the event window is 5 days before splits/reverse splits from January 2010 through November 2011. The regression specification is:

$$Herfindahl\ Index_{i,t,j} = u_{i,t} + \gamma_{i,j} + \rho \times D_{trt_{i,t,j}} + \theta \times return_{i,t,j} + \epsilon_{i,t,j}$$

where i indexes the underlying index, j indexes ETF and t indexes time. The dependent variable in is the Herfindahl Index. $u_{i,t}$ is the index-by-time fixed effects and $\gamma_{i,j}$ is the ETF fixed effects for ETF j of index i. The treatment dummy D_{trt}, equals 0 for the control group. For the treatment group, D_{trt} equals 0 before splits/reverse splits and 1 after splits/reverse splits. $Return_{i,t,j}$ is the return on ETF j of index i on day t. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Dep. Variable	*Herfindahl Index*	
	Split	*Reverse Split*
D_{trt}	-0.047**	0.014*
	(0.02)	(0.01)
return	-0.180	0.040
	(0.13)	(0.05)
Constant	0.272***	0.276***
	(0.01)	(0.00)
R^2	0.734	0.815
N	100	460

Tick Size Constraints, Two-Sided Markets, and Competition Between Stock Exchanges

Yong Chao

University of Louisville

Chen Yao

University of Warwick

Mao Ye

University of Illinois, Urbana-Champaign

Motivation

Continuous price

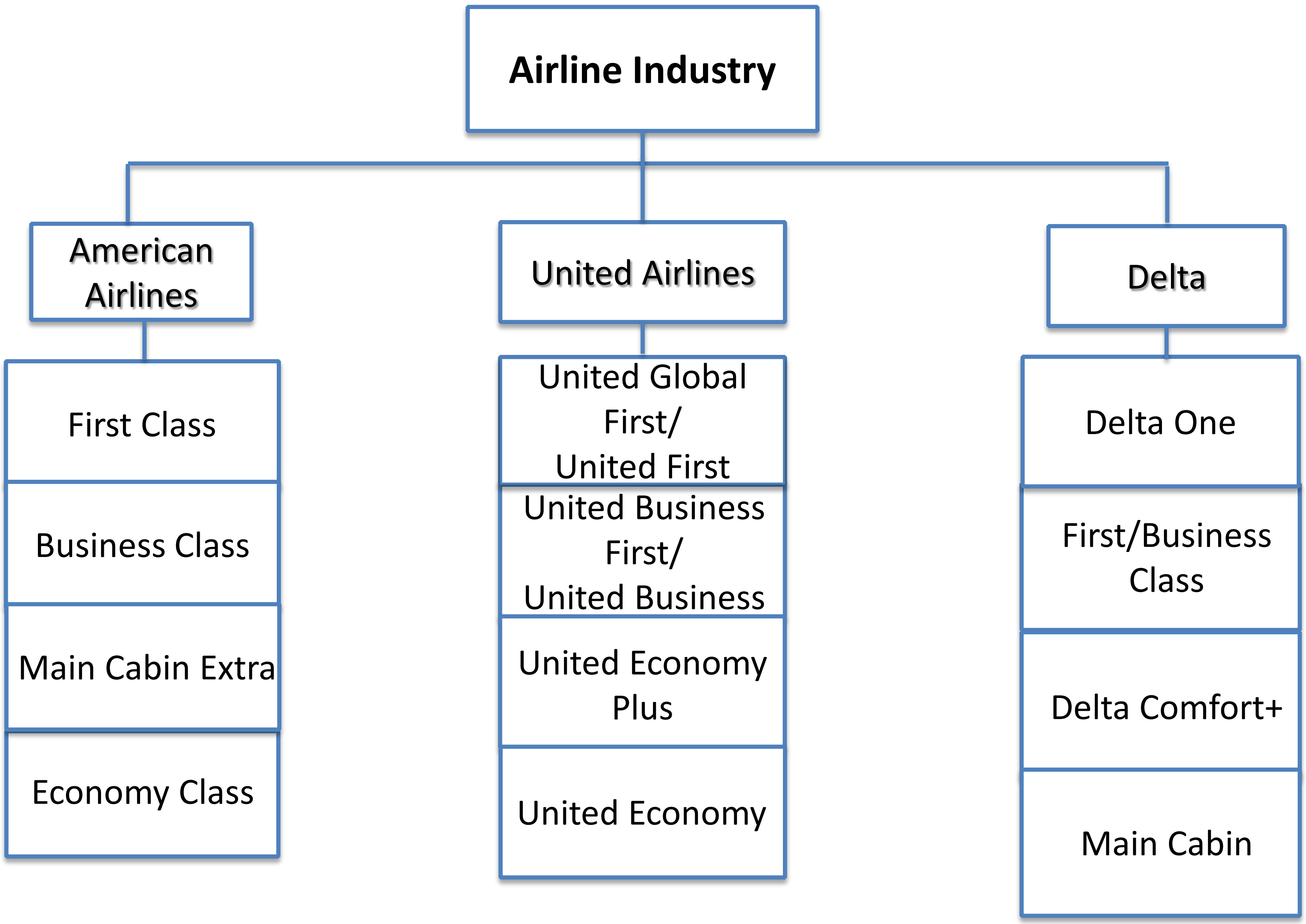
- Implicit assumption in most equilibrium models

Discrete price

- The reality in security trading
- SEC rule 612: 1-cent uniform tick size
  - The minimum price variation is 1 cent for stocks above $1

Tiny levels of friction have dramatic effects

- Impact price competition between stock exchanges
- Influence organization of the exchange industry


Airline Industry
American Airlines
United Airlines
Delta
First Class
Business Class
Main Cabin Extra
Economy Class
United Global First/
United First
United Business First/
United Business
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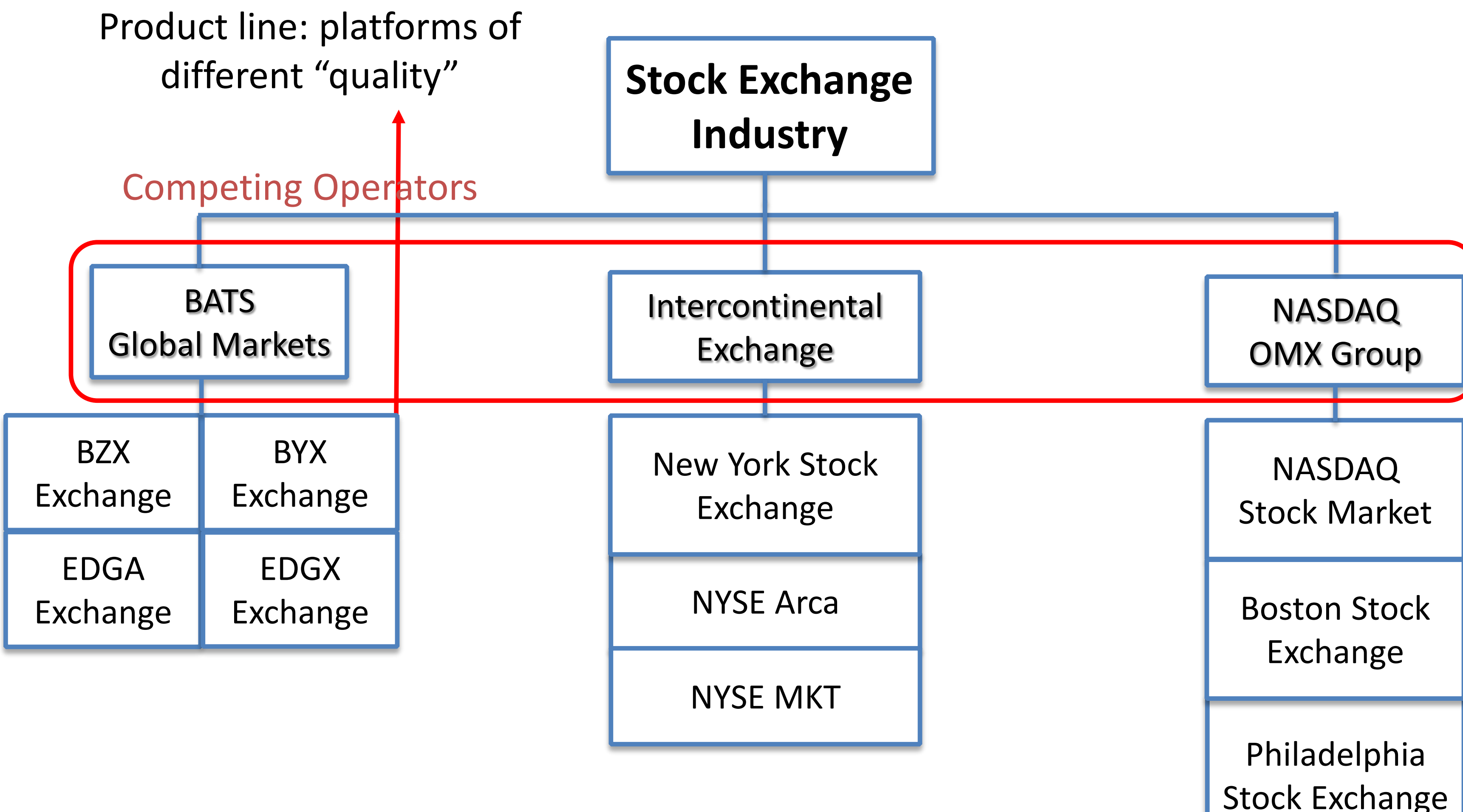
United Economy
Delta One
First/Business Class
Delta Comfort+
Main Cabin



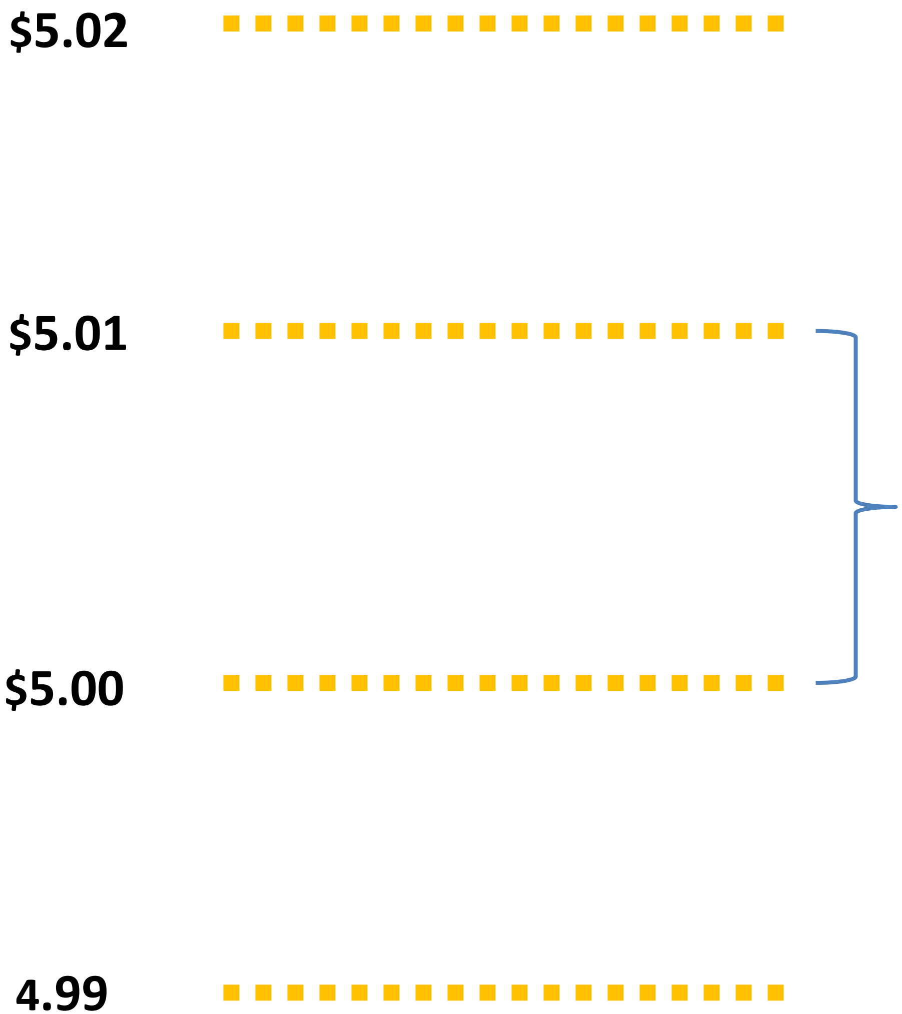
A stock can be traded on any competing platform

- U.S. regulation allows stock trading outside the listing venue

Identical products: limit order books (trading platforms)

Identical pricing models: make-take fee models

Limit Order Book and Tick Size



Tick Size:
minimum price increment
(1 cent here)

Relative Tick Size:
1 cent/Price
(about 20 bps here)

How Trade Occurs

$5.02

Sam: Buys 100 shares 

Mao: Sells 100 shares 

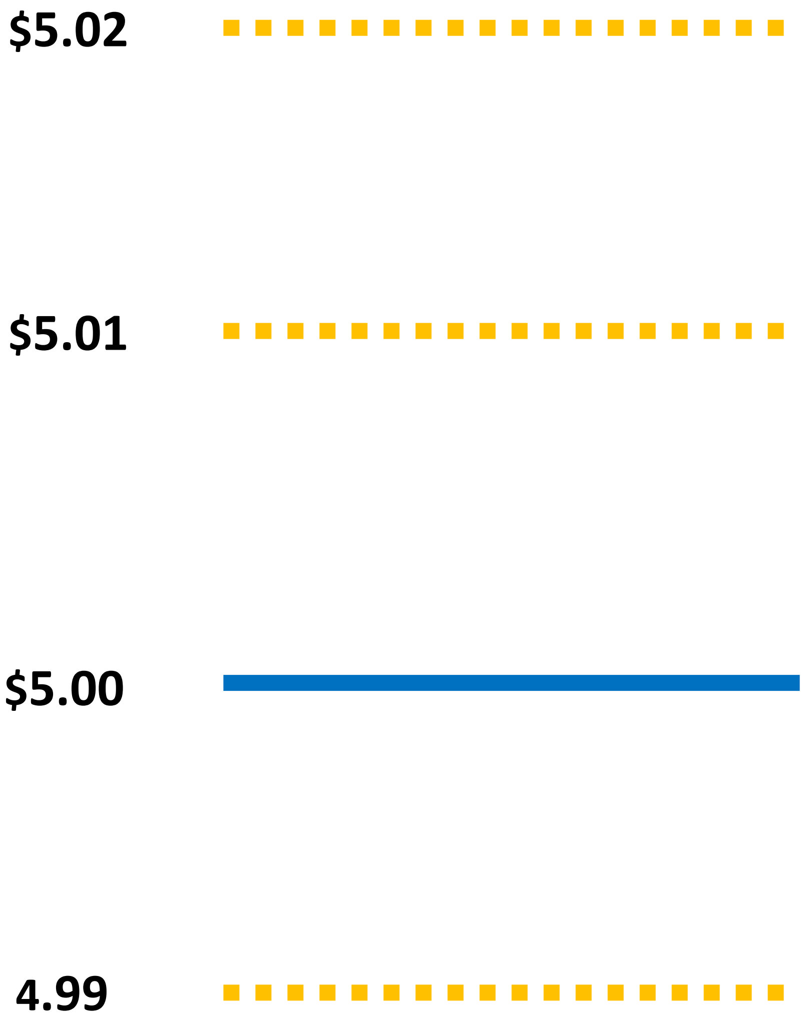
$5.01

$5.00

Sam is a maker, who submits a limit buy order at price 5

4.99

Taker: Market Order Submitter



Sam: Buys 100 shares

Mao: Sells 100 shares

Mao is a taker who places a market order to accept the price of the existing limit buy order

The Profit of the Exchange Per Share

$5.02

Sam (maker) pays a make fee
(Cum fee) effective buy price
= 5.00 + **maker fee**

$5.01

Mao (taker) pays a take fee
(Cum fee) effective sell price
= 5.00 - **taker fee**

$5.00

Total fee = maker fee + taker fee

- Profit of the exchange
- Usually sub-penny per share

4.99

Major source of exchange profit

- Eg. 70% of total profit for BATS

Research Questions

Pricing: how do exchange operators compete?

- Number of platforms to establish
- Make and take fees in each platform

Organization of the industry

- Will an operator have incentives to create multiple platforms?
- Will trading consolidate to a single platform?

Answer: tick size

- Benchmark model: continuous price
  - Does not match stylized facts in exchange competition
- Discrete tick size model
  - Matches stylized facts

Model Timeline

Period 1

Operator(s) choose simultaneously:

- How many trading platforms to offer
- Make fee f_m^i and take fee f_t^i in *each* platform

Period 2

A maker (buyer) with valuation $v_b \sim U\left[\frac{d}{2}, d\right]$ chooses:

- Which platform to submit a limit buy order of one share
- Limit buy order price P_i
  - Effective (cum fee) buy price: $P_i + f_m^i$

Period 3

A taker (seller) with valuation $v_s \sim U\left[0, \frac{d}{2}\right]$ arrives

- Accepts the limit buy order if $P_i - f_t^i > v_s$
  - Effective (cum fee) sell price: $P_i - f_t^i$
- Platform i collects $f_m^i + f_t^i$ conditional on execution

Benchmark Model: Continuous Price

Neutrality of the fees

- The economic outcome depends only on the total fee $T_i = f_m^i + f_t^i$
- Principle: for a given level of tax , it does not matter which side is taxed
  - Traders can choose prices that perfectly counteract any division of the fee

Homogenous platforms compete on the total fee

- No product differentiation

No price discrimination

- No operator has incentives to establish multiple trading platforms
- The platform with the lowest total fee attracts all traders

A Bertrand game in total fee

- Pure strategy equilibrium with zero total fee and profit

Implication for market structure: consolidation

- Trading consolidates to the platform with the lowest total fee

Deviation 1: Non-neutrality

Exchange operators establish multiple platforms with the same total fee but different make-take fee breakdowns

No major operators ever charge both sides (Cardella et al, 2013)

- Some subsidize makers (maker/taker model)
- Some subsidize takers (taker/maker model)
- Others alternate between subsidizing makers and subsidizing takers

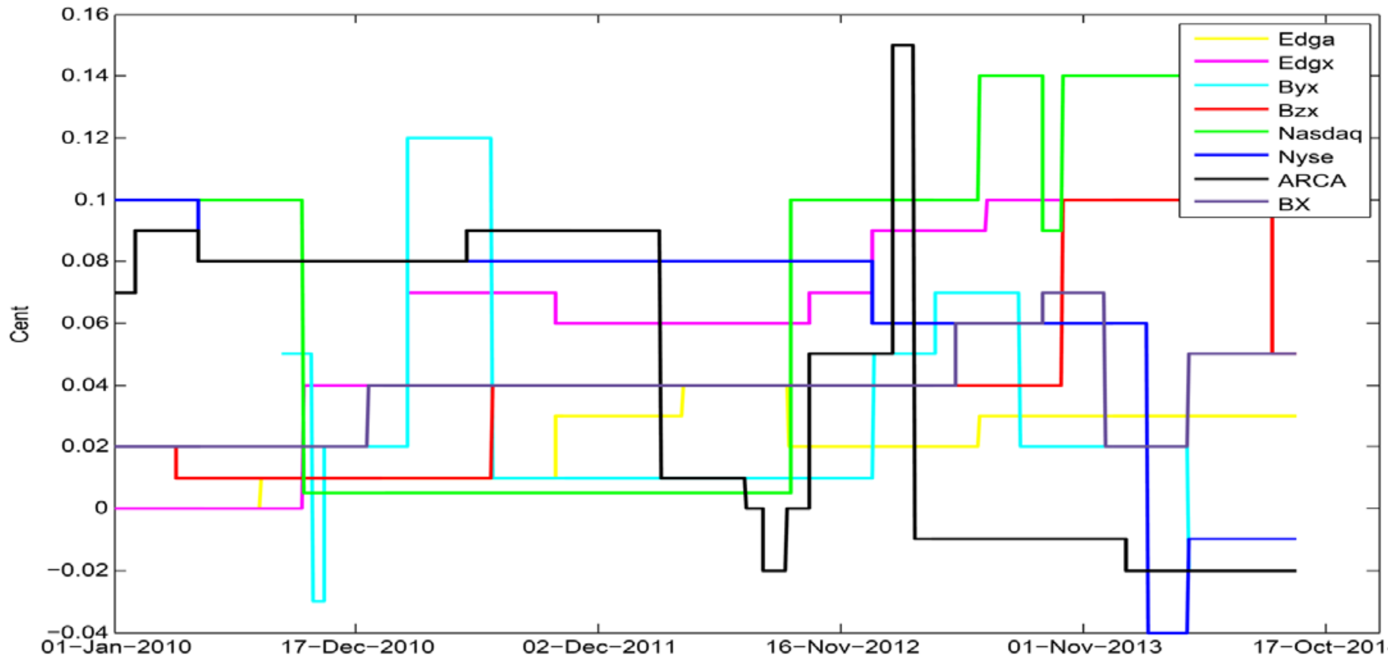
Initiative of regulators to ban the fees based on “fairness”

- Wealth transfer of billions of dollars between makers and takers

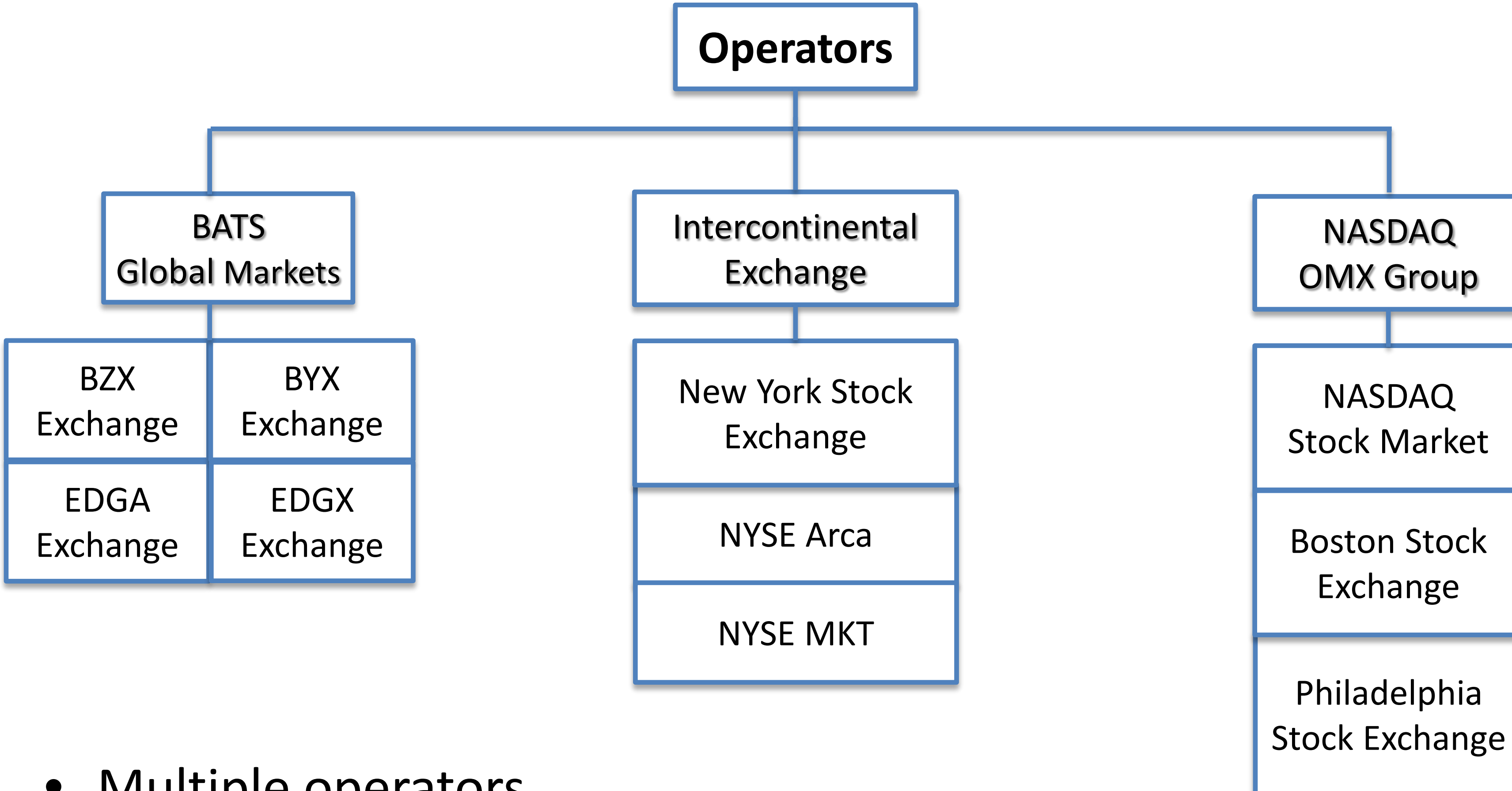
Deviation 2: Non-Bertrand

- Total fee does not converge to a stable value
  - Frequent changes of the make-take fees
  - Initiative of regulators to ban the fees based on their "complexity"

- Entry of platforms with new fee structures



Deviation 3: Market Fragmentation



- Multiple operators
- Each operator creates multiple platforms

Explanation: Discrete Tick Size

Tick size constraints: makers and takers cannot quote sub-penny

- Prevent makers and takers from neutralizing sub-penny fee
  - Change exchange price competition from one-dimension (total fee) to two-dimension (make and take fees)

Two-sidedness creates vertical product differentiation

- For a maker, a platform with better terms for the taker is of high quality
  - Higher execution probability (the taker is more likely to accept the offer)

Product differentiation

- Second degree price discrimination
  - Menu pricing: platforms differentiated by the gains from trade for the maker (implied by make fees) and execution probabilities (implied by take fees)
- Simultaneous choice of price and quality destroys pure strategy equilibrium
  - Mixed strategy equilibria have positive profits

Contributions

Market fragmentation literature

- Market fragmentation
  - Affects liquidity and price discovery (O’Hara and Ye (2011))
  - Leads to mechanical arbitrage opportunities for high frequency traders (Budish, et al (2015), Foucault et al (2015))
  - One driver for systemic risk such as “Flash Crash” (Madhavan (2011))
- A fundamental puzzle: why is stock trading fragmented?

Industrial organization literature

- Two sided platforms (Tirole and Rochet (2006))
- We show that whether end users can neutralize the fees set by platforms affects
  - Price competition between platforms
  - Structure of the industry

Provide insights to two recent policy initiatives

- Increasing the tick size to 5 cents
- Banning make-take fees

Road Map

1. Nature of Fee Game and Non-neutrality
2. Vertical Product Differentiation
3. Second Degree Price Discrimination
4. Competing Operators and Non-Bertrand
5. Conclusion and Policy Implications

A Simple Example with Exogenous Fees

Tick size: 1

A maker arrives at period 2 who

- Has valuation $v_b \sim U[0.5,1]$
- Intends to buy
- Submits a limit order of one share at $P = \{0, 1\}$ or no order at all

A taker arrives at period 3 who

- Has valuation $v_s \sim U[0, 0.5]$
- Intends to sell
- Decides whether to accept maker's offer

Who Cares About Tick Size and Fees ?

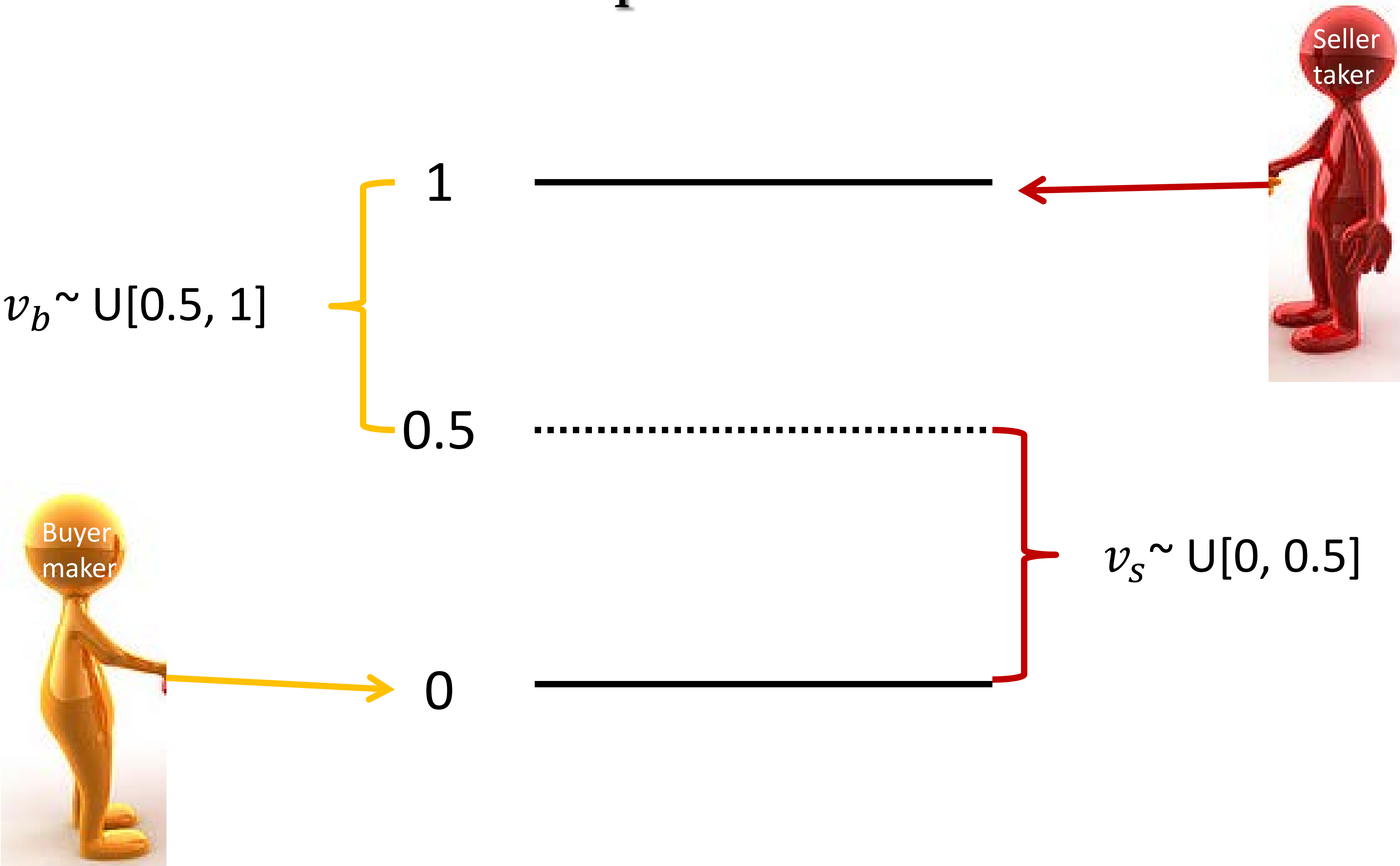
High-frequency traders (Yao and Ye, 2015)

Agency brokers who aim to minimize transaction costs for institutional investors

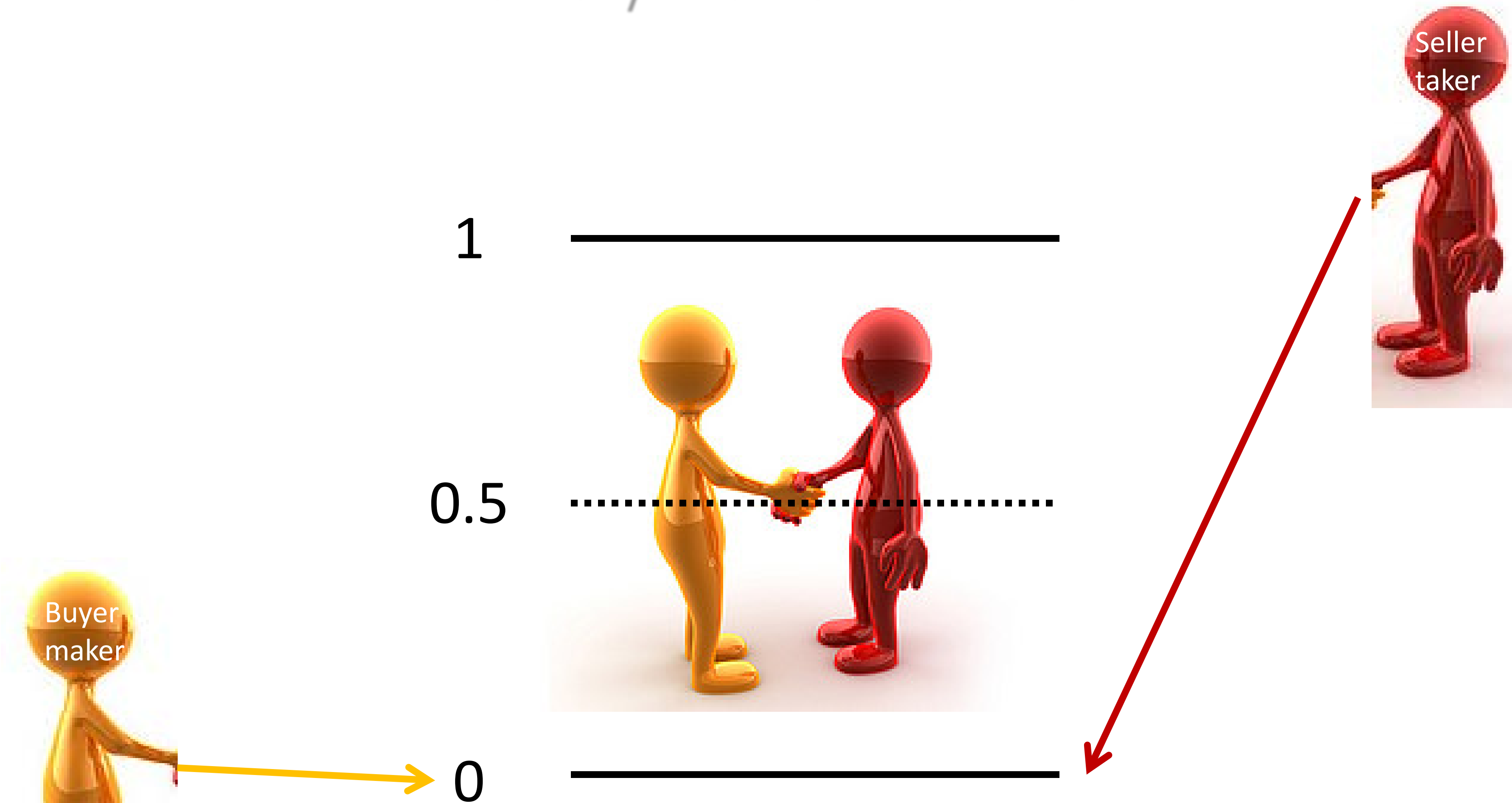
Brokers for retail traders

- Battalio, Corwin and Jennings (2014)
    - Brokers can pocket rebates (negative fees) and not pass them to retail traders

No Fees: Tick Size Leads to No-Trade Equilibrium



Taker/maker Market



Submit limit buy order at 0 and pay 0.5 fee (effective buy price = 0.5)

Sell at 0 and pay -0.5 fee (effective sell price = 0.5)

Non-Neutrality

Zero total fee can lead to no-trade equilibrium

- $f_m = 0$ and $f_t = 0$

Zero total fee can also lead to the socially optimal outcome

- $f_m = 0.5$ and $f_t = -0.5$
- Maker submits a buy order at price $P = 0$
  - Effective buy price $P_b = P + f_m = 0.5$
  - Effective sell price $P_s = P - f_t = 0.5$

Non-Neutrality: fee breakdowns affect market outcome

We show that one side must be subsidized for trades to occur

- Explains why no major platforms ever charge both sides

Nature of the Fee Game

Fees determine the effective buy and sell prices

- $P^i = 0$ if $f_m^i > 0$
  - Effective buy price $P_b^i = f_m^i$
  - Effective sell price $P_b^i = -f_t^i$
- $P^i = 1$ if $f_m^i < 0$
  - Effective buy price $P_b^i = 1 + f_m^i$
  - Effective sell price $P_s^i = 1 - f_t^i$

Platforms compete for order flow through differentiated

- Effective buy price
  - Affects gains from trade for the maker: $\left(v_b - p_b^{\mathrm{i}}\right)$
- Effective sell price
  - Affects the maker's probability of execution: $\tilde{q}_s^i = \Pr\left(v_s \leq p_s^i\right) = 2\,\mathrm{Min}\left\{p_s^i, \frac{1}{2}\right\}$

Two Side Effects of Subsidy

1. Force the maker to quote a more aggressive price
   - The maker can quote a buy price of 0 when $f_m^i > 0$
   - The maker can only quote a buy price of 1 when $f_m^i < 0$
2. Reduce execution probability
   - Higher subsidy to maker comes from higher fee to the taker
   - Reduces the taker's willingness to trade

The side effects are magnified when tick size is larger

Makers prefer positive fee instead of subsidy when tick size is large

- Explains the existence of market charging liquidity makers
- Yao and Ye (2015) find the market charging liquidity makers are more active for low-priced stocks
  - Relative tick size = 1 cent/share price

Competing Story

"It was not obvious to Brad why some exchanges paid you to be a taker and charged you to be a maker, while others charged you to be a taker and paid you to be a maker. No one he asked could explain it, either. To Brad this all just seemed bizarre and unnecessarily complicated—and it raised all sorts of questions. "Why would you pay anyone to be a taker? I mean, who is willing to pay to make a market? Why would anyone do that?"

– Michael Lewis: Flash Boys

Road Map

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Decision under Exogenous (p_b^1, p_s^1) and (p_b^2, p_s^2)

Period 3: seller (taker)

- Effective sell price $p_s^i = P^i - f_t^i$
- Accepts the offer if $v_s \leq p_s^i$
- $\tilde{q}_s^i = \Pr\left(v_s \leq p_s^i\right) = 2\,\mathrm{Min}\left\{p_s^i, \frac{1}{2}\right\}$

Period 2: buyer (maker) proposes a buy limit order at price P

- Effective buy price $p_b^i = P^i + f_m^i$
- Submits a limit buy order to platform i to maximize
  - Expected surplus: $(v_b - p_b^{\mathrm{i}})\tilde{q}_s^i$
- Submits no limit order
  - If v_b<min (p_b^1, p_b^2)

Quality of Platforms Differ

Platform with higher effective sell price p_s^i has higher execution probability

- $2\left(v_b - p_b^{\mathrm{i}}\right) \mathrm{Min}\left\{p_s^i, \frac{1}{2}\right\}$

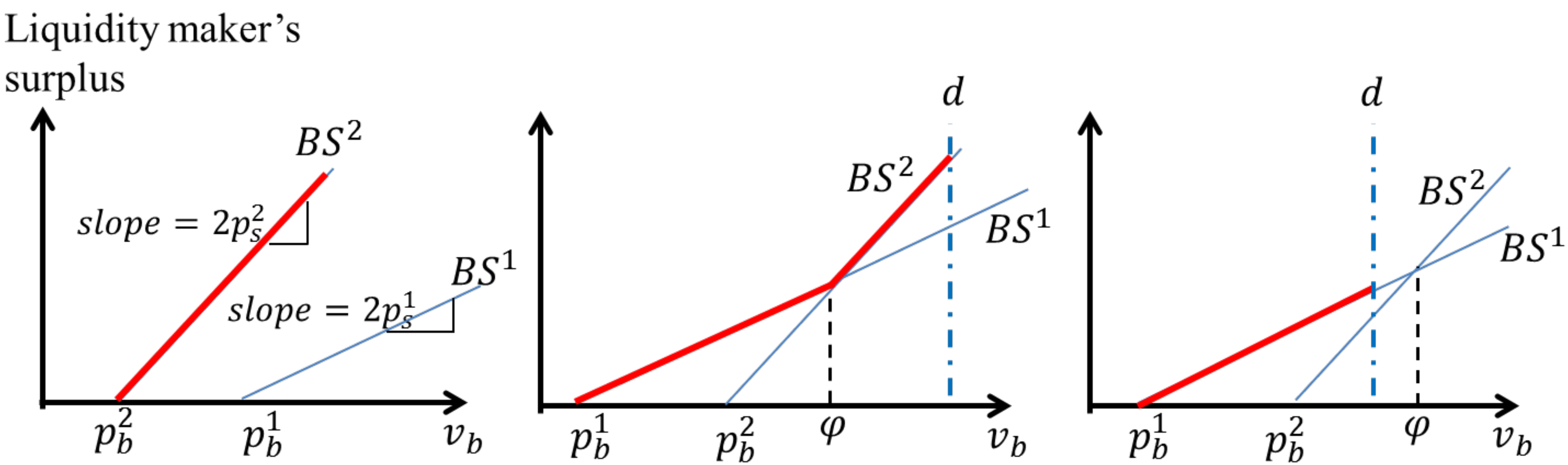
Makers prefer a market with a higher p_s^i for a fixed p_b^i

- Vertical product differentiation
  - Platform with higher execution probability has higher “quality”
  - Makers are more likely to realize gains from trade $\left(v_b - p_b^{\mathrm{i}}\right)$

No product differentiation under continuous price

- All makers prefer the market with the lowest total fee

Maker's Segmentation



(a) $p_b^2 \leq p_b^1$ *Platform 2 Only*

(b) $p_b^1 < p_b^2, \varphi < d$ *Co-exist*

(c) $p_b^1 < p_b^2, \varphi \geq d$ *Platform 1 Only*

$$BS^1 = \left(v_b - p_b^1\right) \cdot 2p_s^1$$

$$BS^2 = \left(v_b - p_b^2\right). 2p_s^2$$

φ: the marginal maker

Road Map

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4. Competing Operators and Non-Bertrand
5. Conclusion and Policy Implications

Benchmark: One Operator, One Platform

$$\max_{p_b, p_s} \pi = (p_b - p_s) \cdot q_b \cdot q_s$$

$$st. q_b = \Pr\left(v_b > \max(p_b, \frac{1}{2})\right)$$

$$q_s = \Pr\left(v_s < \min(p_s, \frac{1}{2})\right)$$

Solution

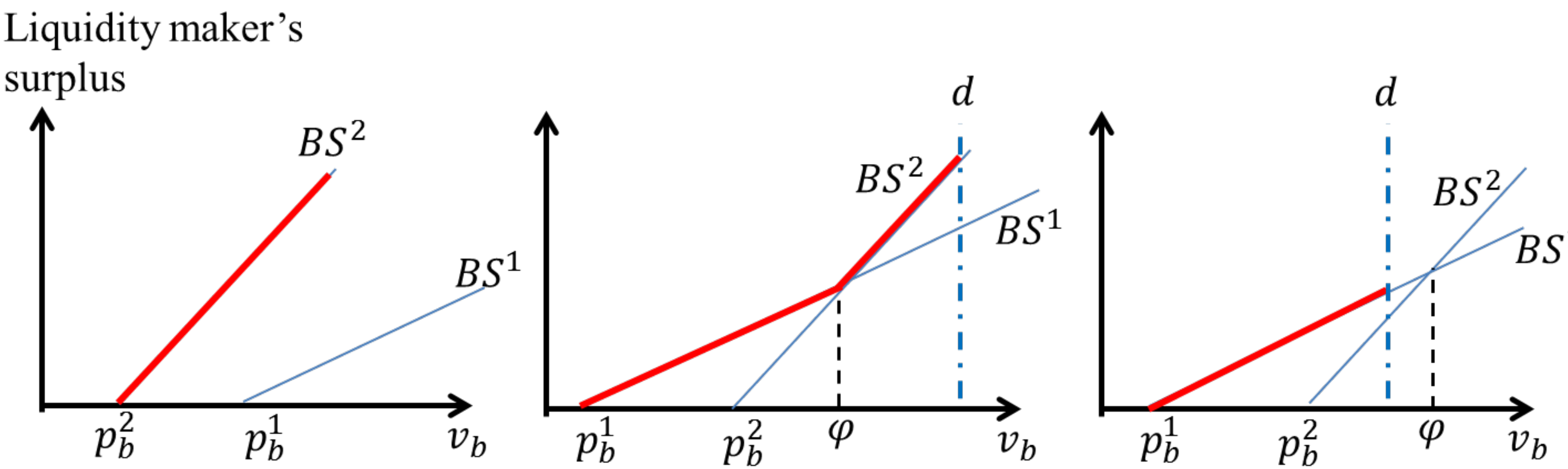
- $p_b = \frac{2}{3}$, $q_b = \Pr\left(v_b > \frac{2}{3}\right) = \frac{2}{3}$
- $p_s = \frac{1}{3}$, $q_s = \Pr\left(v_b < \frac{1}{3}\right) = \frac{2}{3}$
- $p_b - p_s = \frac{1}{3}$

One Operator, Two Platforms

$$\max_{0<p_b^1,p_s^1,p_b^2,p_s^2<1} \pi = \left(p_b^1 - p_s^1\right)\Pr(A_1)\Pr(v_s \le p_s^1)$$

$$+\left(p_b^2 - p_s^2\right)\Pr(A_2)\Pr(v_s \le p_s^2)$$

St. A_i *={Maker with* v_b *chooses platform i)*



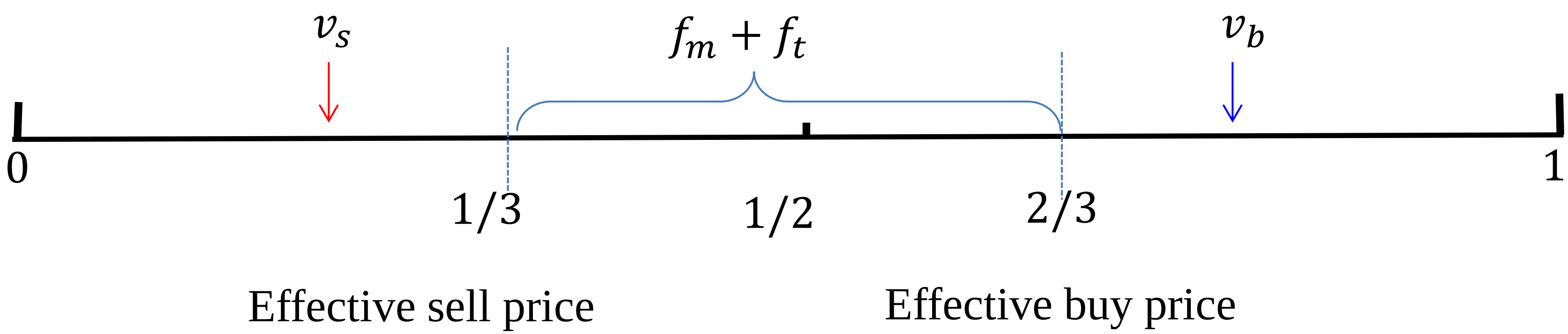
(a) $p_b^2 \le p_b^1$ *Platform 2 Only*

(b) $p_b^1 < p_b^2, \varphi < d$ *Co-exist*

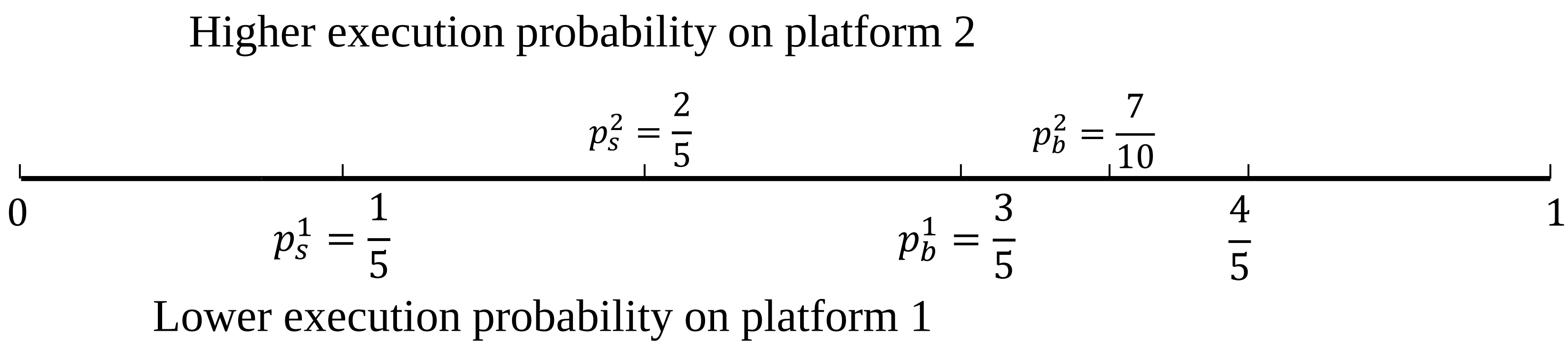
(c) $p_b^1 < p_b^2, \varphi \ge d$ *Platform 1 Only*

Comparison: Price Proposed by Maker

One Platform



Two Platforms



Comparison: Price Proposed by Maker

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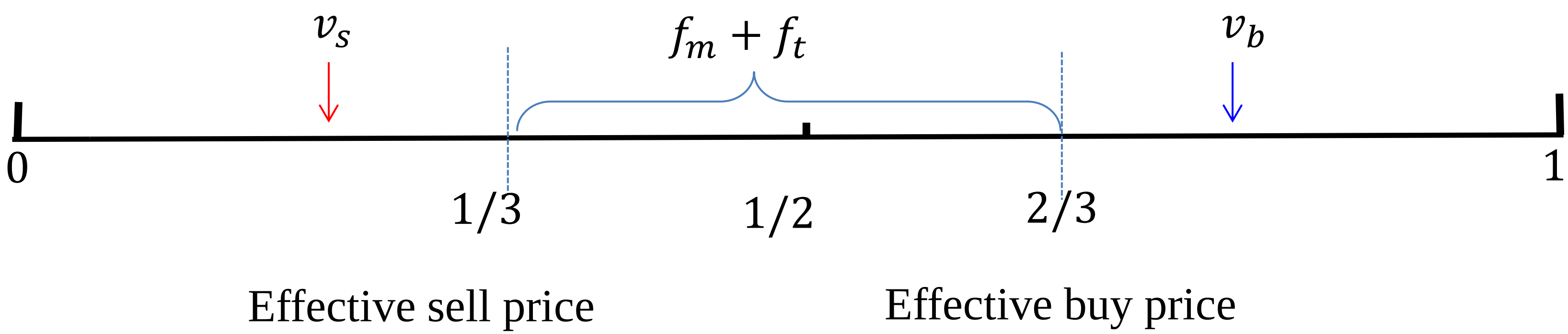
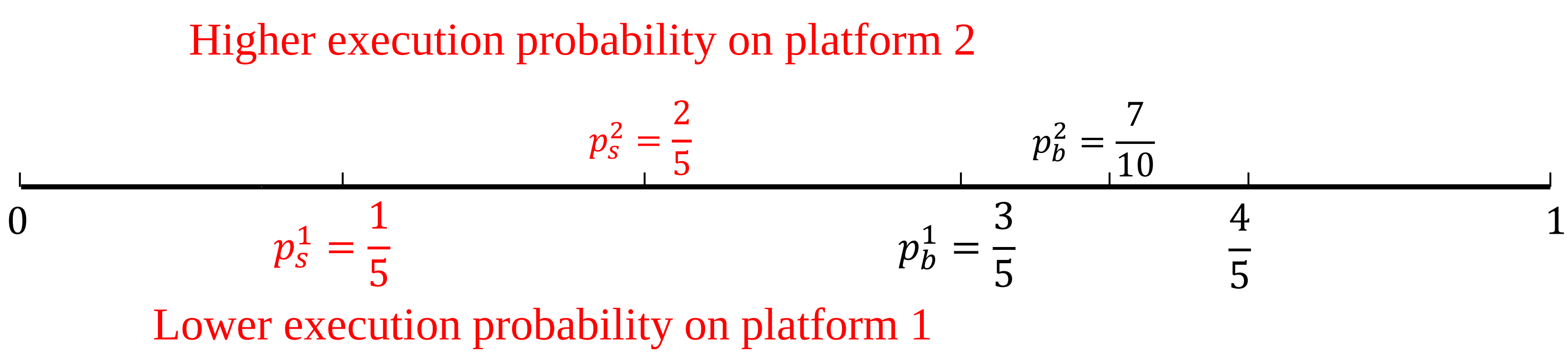
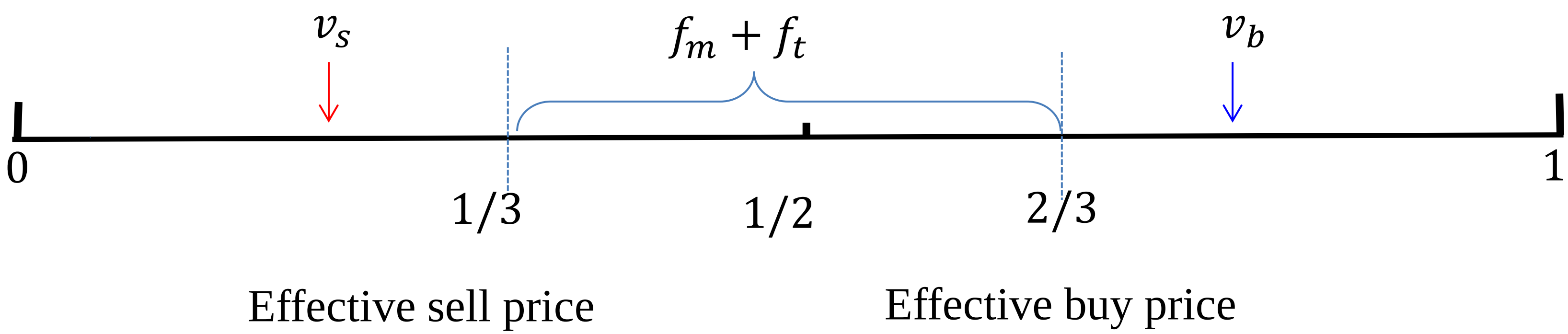


Two Platforms

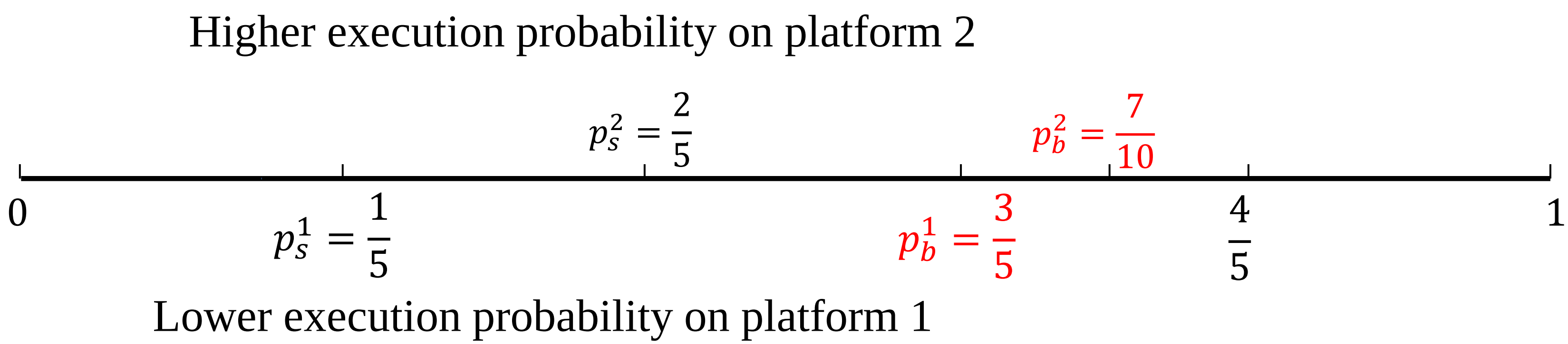


Comparison: Price Proposed by Maker

One Platform

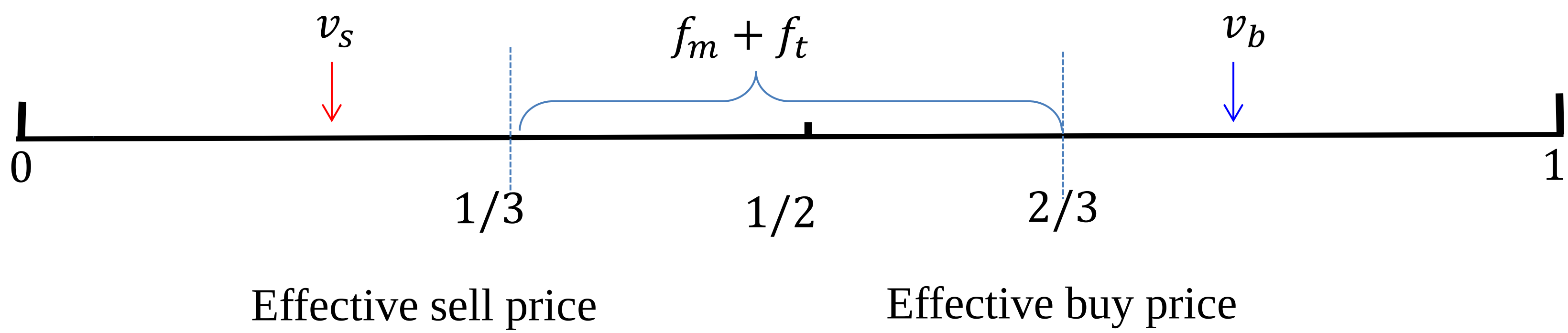


Two Platforms

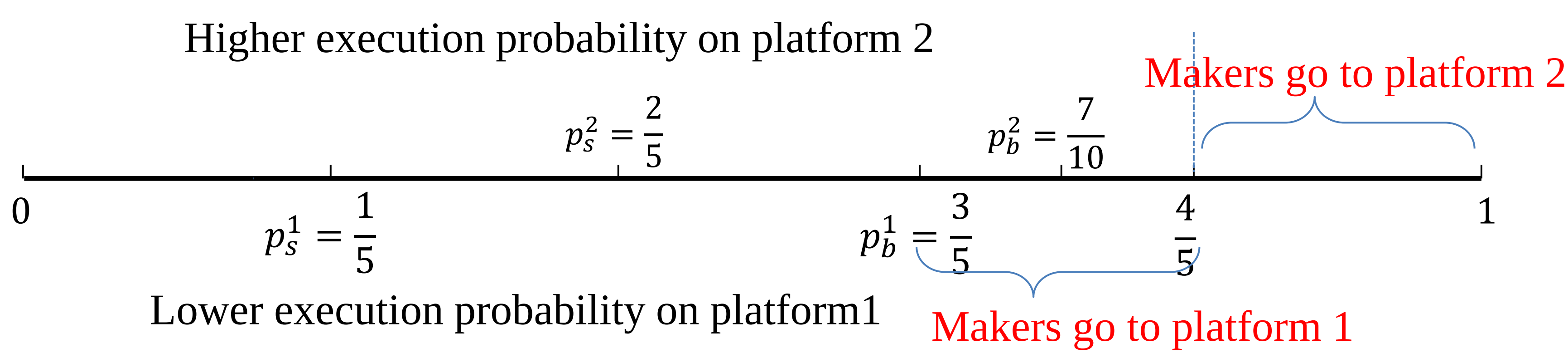


Comparison: Price Proposed by Maker

One Platform



Two Platforms



Comparison: Volume and Welfare

		One Platform	Two Platforms	
			Low Quality	High Quality
Maker Participation		$\frac{2}{3}$	$\frac{2}{5}$	$\frac{2}{5}$
Taker Participation		$\frac{2}{3}$	$\frac{2}{5}$	$\frac{4}{5}$
Volume		$\frac{12}{27}$	$\frac{4}{25}$	$\frac{8}{25}$
			$\frac{12}{25}$	
Welfare	Operator	$\frac{4}{27}$	$\frac{4}{25}$	
	Maker	$\frac{2}{27}$	$\frac{2}{25}$	
	Taker	$\frac{2}{27}$	$\frac{2}{25}$	

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		One Platform	Two Platforms	
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Comparison: Welfare

		One Platform	Two Platforms	
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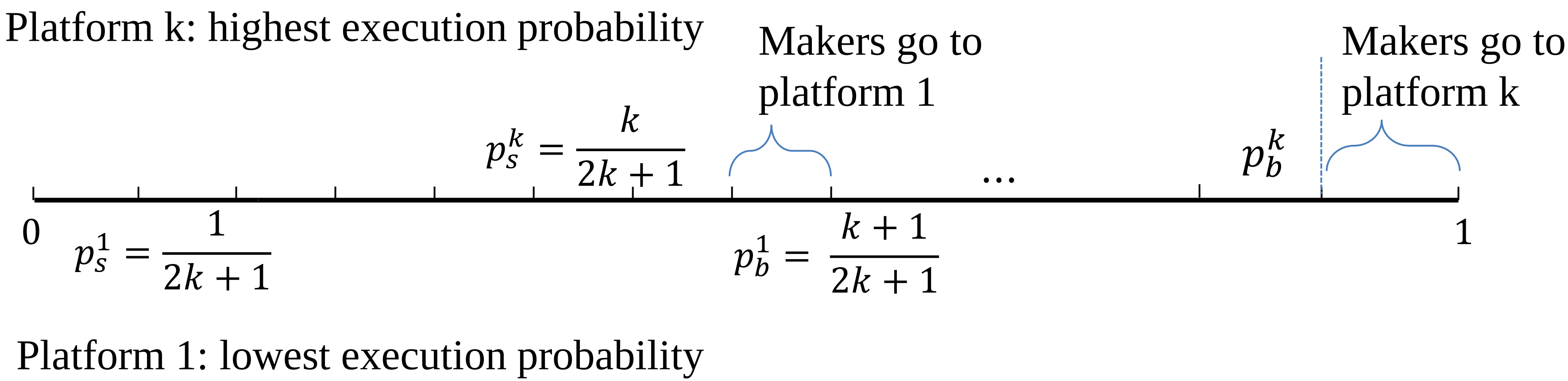
	Maker	$\frac{2}{27}$	$\frac{2}{25}$	
	Taker	$\frac{2}{27}$	$\frac{2}{25}$	

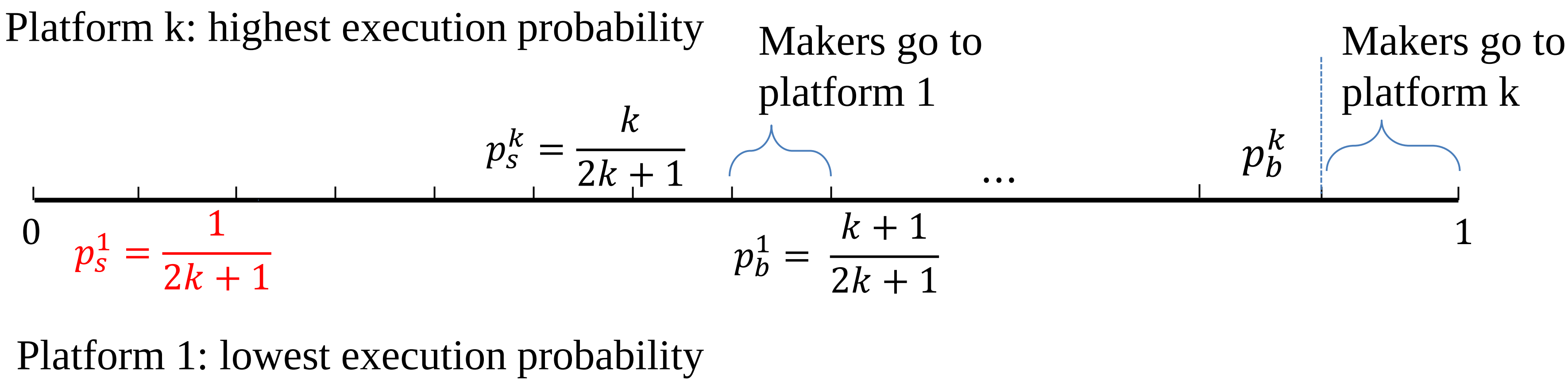
One Operator, k Platforms



- As *k* increases:
  - Welfare for all participants (maker, taker, operator) increases
  - Marginal benefit of opening one more platform decreases

- Number of platforms under a fixed cost c
  - $\bar{k} = \max\left\{k \in \mathbb{N} \middle| \frac{4k}{3(2k+1)^2(2k-1)^2} \geq c\right\}$

One Operator, k Platforms



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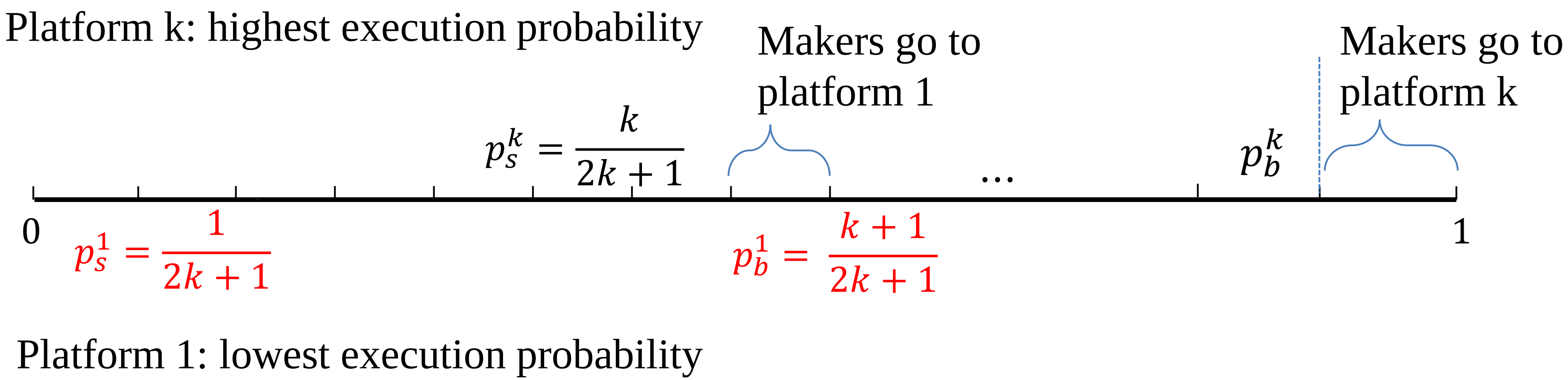
One Operator, k Platforms



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A New Form of Price Discrimination

Puzzle: how does fee differentiation create price discrimination?

- Empirical Evidence: NASDAQ and NASDAQ BX (Skjeltorp et al (2011))
- *"it is not clear however how the differentiation of make/take fees suffices to screen different types of investors since, in contrast to payments for order flow, liquidity rebates are usually not contingent on investors' characteristics (e.g., whether the investor is a retail investor or an institution)"* (Foucault (2012))

This paper shows

- Operator uses take fees to price discriminate against makers
  - A new form of second degree price discrimination
- Price discrimination using fees can improve welfare

Road Map

1. Nature of Fee Game and Non-neutrality
2. Vertical Product Differentiation
3. Second Degree Price Discrimination
4. Competing Operators and Non-Bertrand
5. Conclusion and Policy Implications

Two Competing Operators

Each operator offers one platform

- Maximizes profit conditional on competitor's strategy

Simultaneously choose effective buy and sell price

- For the maker: simultaneous choice of price and quality

Result

- Non-existence of pure strategy equilibrium
  - Explains fee diversity
  - Explains the frequent changes of fee structures
- Mixed strategy equilibria have positive profit
  - Explains entry of platforms with new fee structures

Non-existence of PNE with Positive Profit

Intuition follows Bertrand:

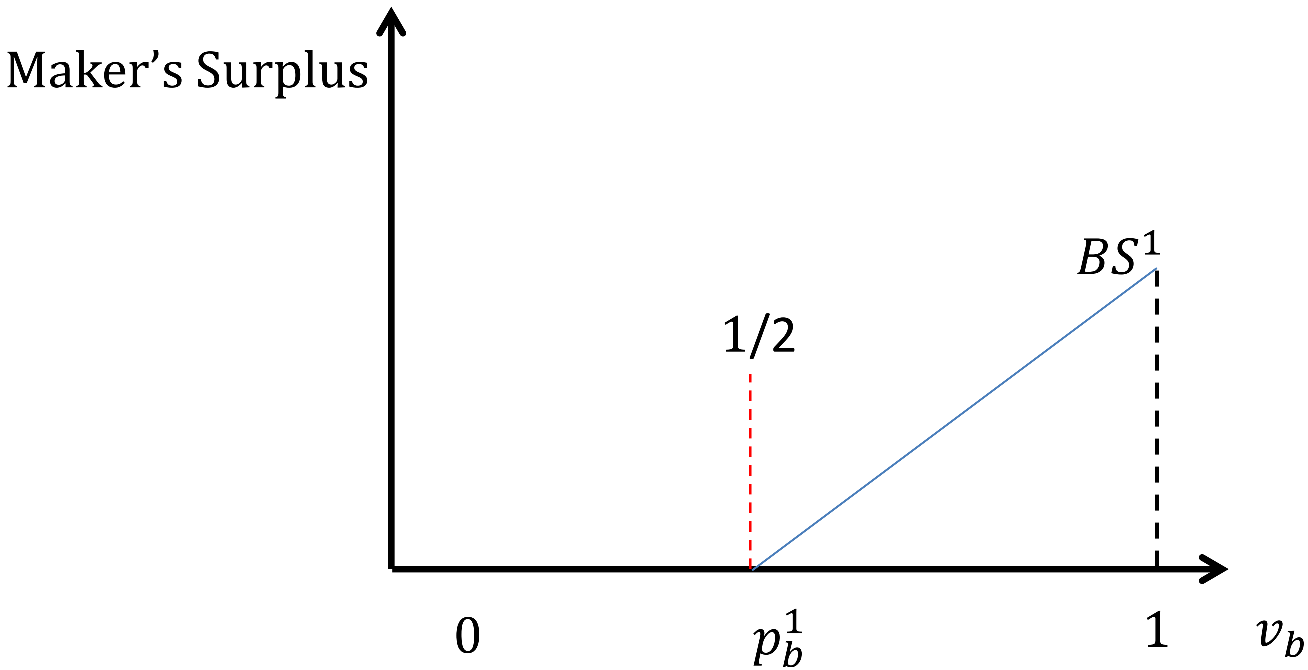
- Consider operator 2's profitable deviation
- Suppose the profit of operator 1 $\geq$ operator 2
  - Operator 2 obtains the whole market if he:
    - Undercuts operator 1's effective buy price by ε
    - Sets the same effective sell price

Non-existence of PNE with Zero Profit

Consider operator 2's profitable deviations in each of the following cases:

- Trivial case: no trade
- $p_b^1 = p_s^1 > \frac{1}{2}$
  - Operator 2: $p_s^2 = \frac{1}{2}$ and $p_b^2 = p_b^1 - \varepsilon$
    - $p_b^2 - p_s^2 > 0$
    - Execution probability on platform 2 equals to 1
    - Lower effective buy price attracts all makers
- $p_b^1 = p_s^1 = \frac{1}{2}$ (to be discussed)
- $p_b^1 = p_s^1 < \frac{1}{2}$ (to be discussed)

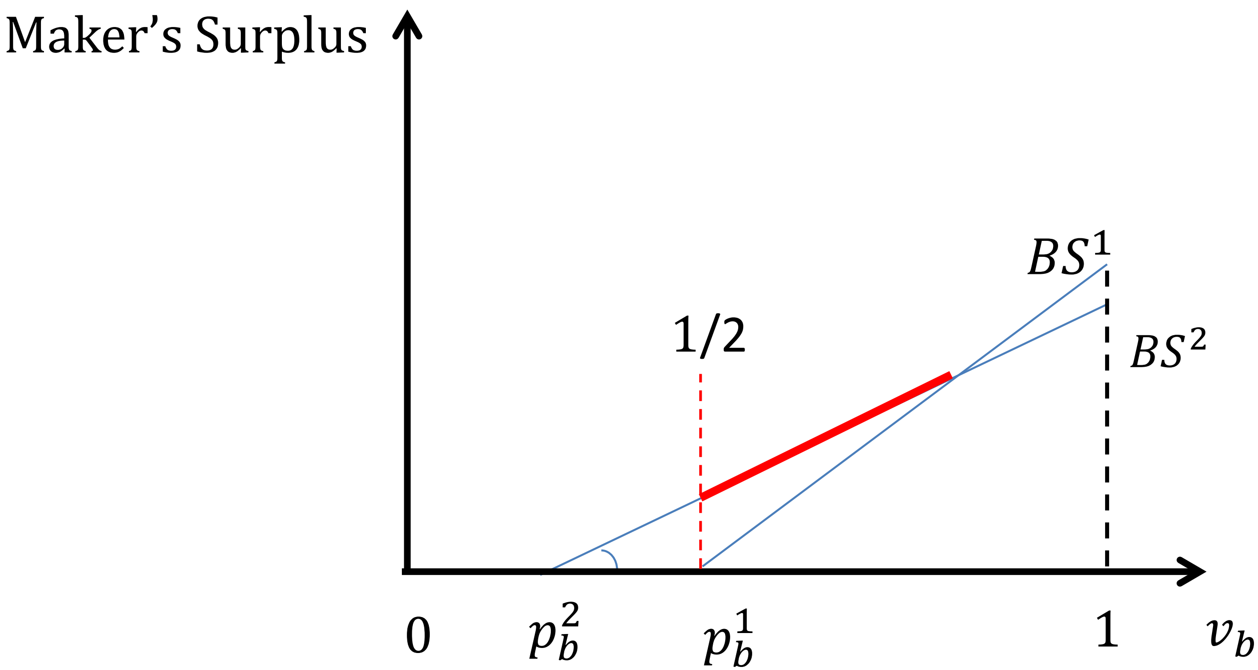
Operator 1: $p_b^1 = p_s^1 = \frac{1}{2}$



$$BS^1 = \left(v_b - p_b^1\right) \cdot 2p_s^1$$
$$= v_b - 1/2$$

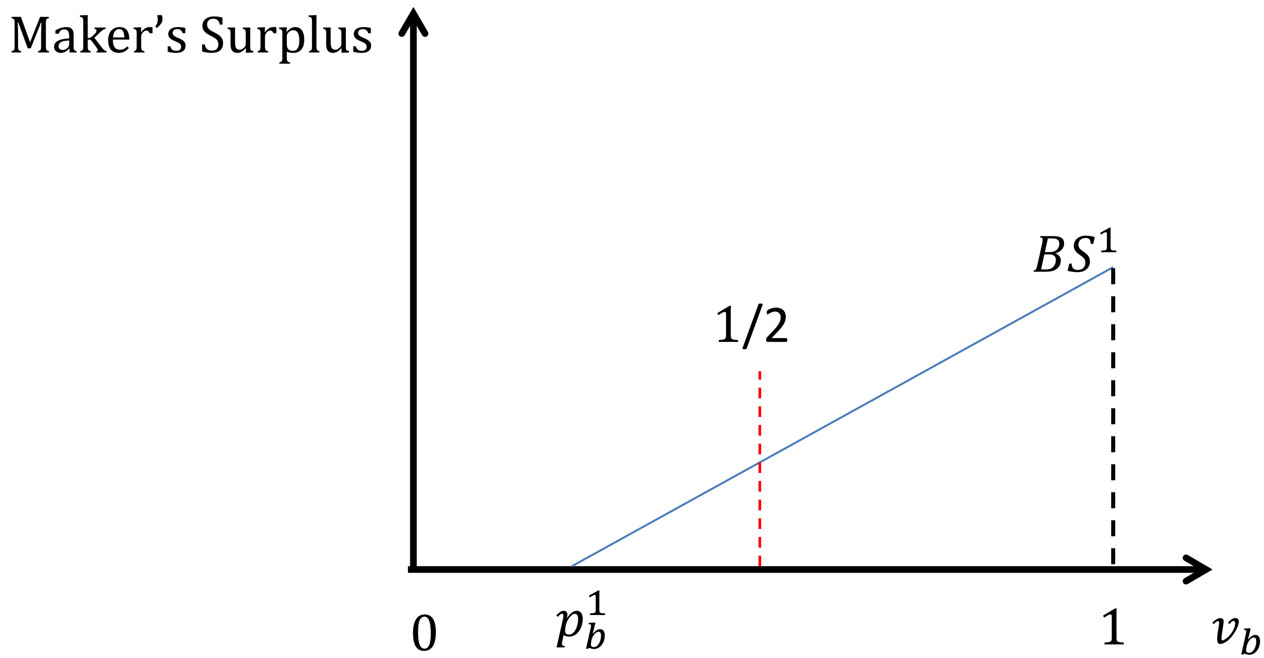
Deviation: Reduces Price and Quality

$$p_b^2 = \frac{1}{2} - \mu\varepsilon, p_s^2 = \frac{1}{2} - \varepsilon \text{ and } 0 < \mu < 1$$



- Operator 2 attracts makers with lower gains from trade
  - $p_b^2 - p_s^2 > 0$, which leads to positive profit

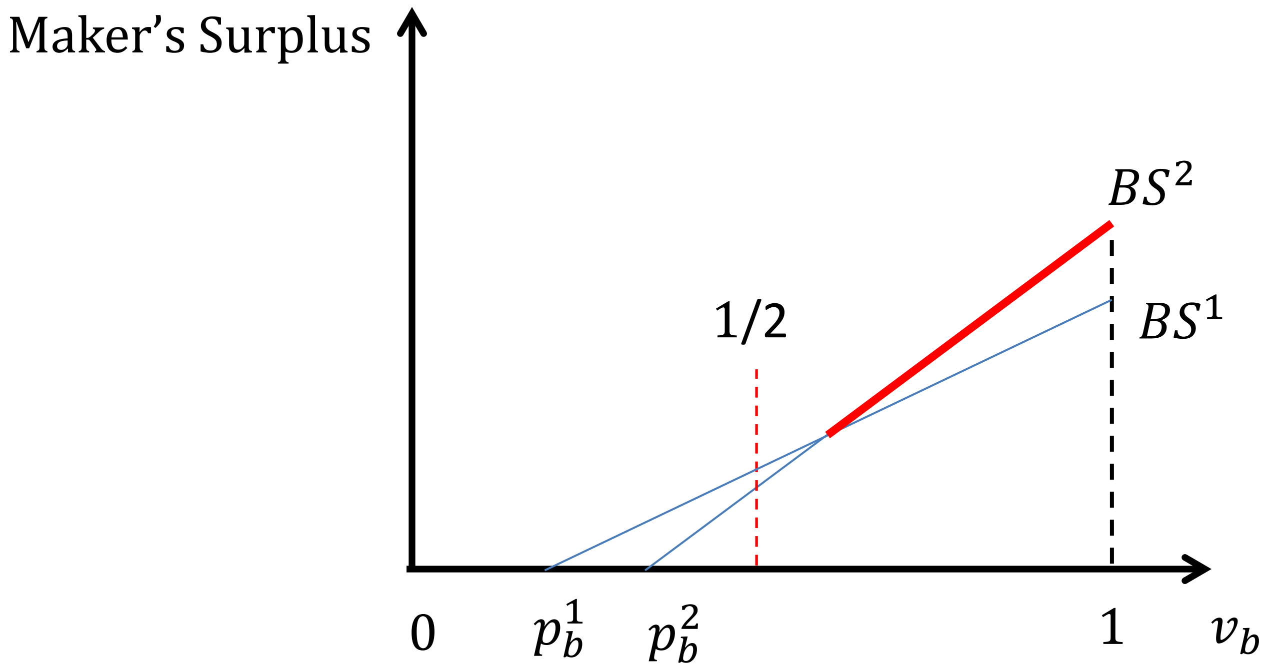
Operator 1: $p_b^1 = p_s^1 < \frac{1}{2}$



$$BS^1 = \left(v_b - p_b^1\right) \cdot 2p_s^1$$

Deviation: Increases Price and Quality

$$p_b^2 = p_b^1 + \varepsilon, p_s^2 = p_s^1 + \mu\varepsilon \text{ and } 0 < \mu < 1$$



- Operator 2 attracts makers with higher gains from trade
  - $p_b^2 - p_s^2 > 0$, which leads to positive profit

An Atypical Price-Quality Game

Most price-quality games have pure strategy equilibrium

- Quality is determined before price
- Quality is a long-term decision

Exchange industry: product quality for the maker can be adjusted with relatively low friction

- Simultaneous decision of price and quality generates mixed strategy (Chioveanu, 2012)

Our model has typical pure strategy equilibrium if

- Operators set the take fees in the first stage
- Operators set the make fees after observing each others' take fees in the second stage

Mixed Strategy Equilibria

Mixed strategy

- Explains frequent fee structure changes
- Explains the diversity of fee structures

Positive profit

- Explains entry of new platforms

One Set of Mixed-strategy Equilibria

Corollary 2: *One set of symmetric mixed-strategy equilibrium is as follows.*

(a) $p_b - p_s = \frac{1}{6} \cdot d$;

(b) p_b *is randomized over* $[L, U] \subset [\frac{1}{2}, \frac{7}{12}]$ *with distribution function* $F(x)$*, where*

$$F(x) = \frac{C_1}{\left(\frac{9}{2}\right)^{\frac{1}{3}} \cdot \left(x - \frac{4}{9}\right)^{\frac{1}{3}}} + \frac{C_2}{90 \cdot \left(x - \frac{1}{6}\right) \cdot 2^{\frac{1}{3}}} \cdot \left[1 - H \cdot \left(\frac{x - \frac{1}{6}}{x - \frac{4}{9}}\right)^{\frac{1}{3}}\right] \quad (25)$$

Here H *is a Hypergeometric function* $F_1\left(\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{5}{18\left(x - \frac{1}{6}\right)}\right)$*, and* (C_1, C_2, L, U) *satisfy:*

$$\int_x^U t \cdot dF(t) - \int_L^x t \cdot dF(t) + \left(\frac{3}{2} - 2x\right) \cdot F(x) + \left(x - \frac{1}{6}\right)\left(\frac{4}{3} - 3x\right) \cdot F'(x) = \frac{1}{6} \quad (26)$$

$$F(L) = 0 \quad (27)$$

$$F(U) = 1 \quad (28)$$

$$F'(x) > 0 \text{ for any } x \in [L, U] \quad (29)$$

Conclusion

Discrete tick size leads to non-neutrality of the fees

Non-neutrality allows exchange operators to create differentiated products

- Leads to second degree price discrimination
- Destroys any pure strategy equilibrium

Discrete tick size leads to market fragmentation

- Each exchange operator has incentive to open multiple platforms
- Competition between operators leads to mixed strategy equilibria with positive profits

Policy Implications I

Proposals by regulators to ban the fee

- Complex: diversity and frequent changes of fee structures
  - Explained by mixed strategy equilibria
- Unfair: billions of dollars of wealth transfer
  - Our paper shows that fees can improve social welfare
  - The side being charged can also benefit
  - Makers may even prefer being charged instead of being subsidized

Proposals by regulators to increase the tick size to 5 cents

- Main goal: to increase the number of IPOs
- Our paper implies that it can lead to fee competition between exchanges and even more trading fragmentation

Make-take fees and market fragmentation are market design responses to existing regulations!

Policy Implications II

- Partial equilibrium regulation
  - Policy initiatives on tick size, make-take fee, and market fragmentation are often discussed in isolation
  - We should be cautious of the partial equilibrium regulation because changing one may affect others

- Additional regulation vs. deregulation
  - Deregulate the tick size?
  - In order to pursue additional regulation, we must first evaluate current regulation