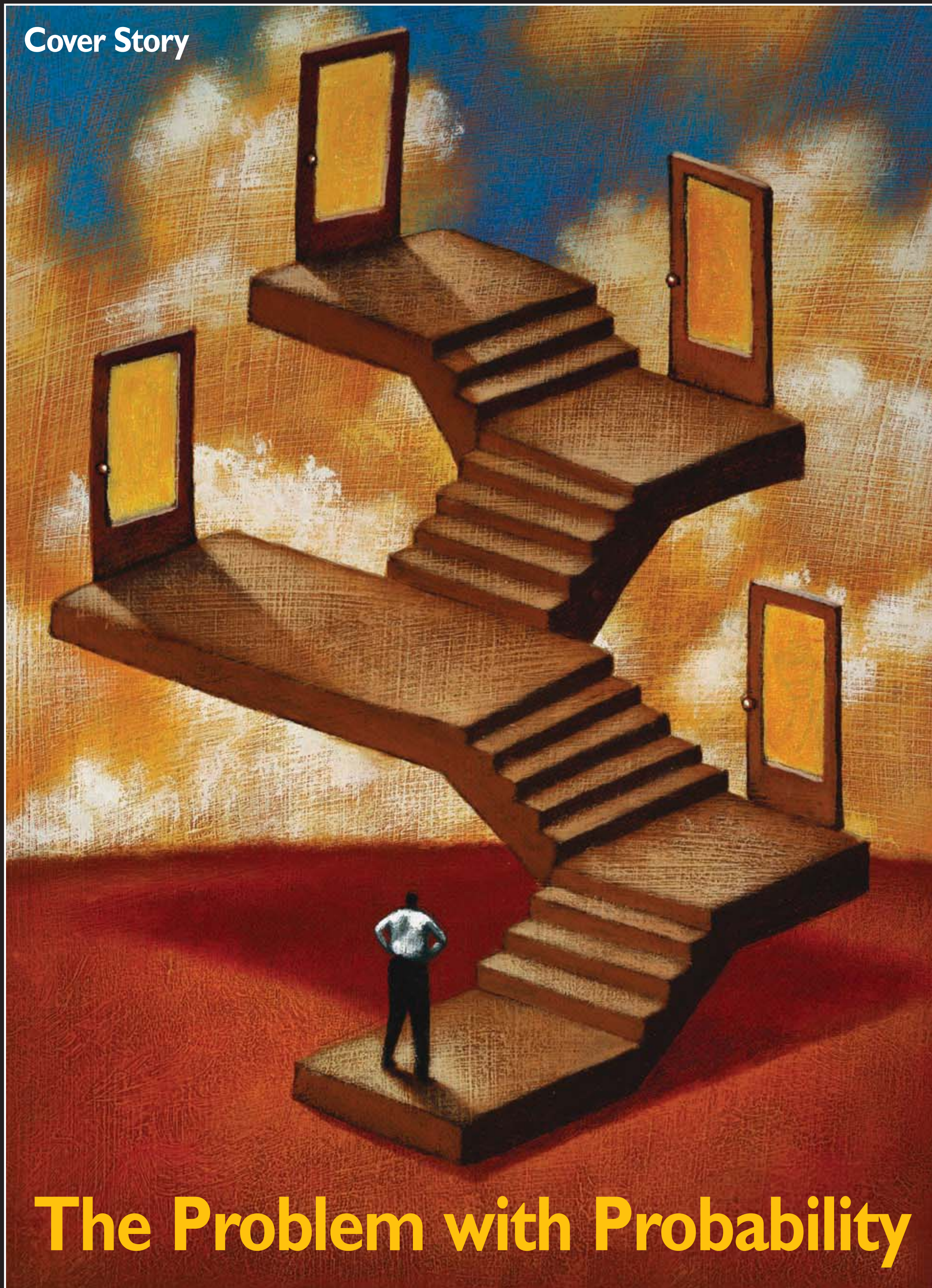
Cover Story



The Problem with Probability

In the US, to comply with multiple financial reporting standards, all public corporations must use probability to perform fair value calculations. However, the existing probability requirements lack formality and are too opaque. Gordon E. Goodman offers examples of the inconsistent application of probability across different types of financial transactions and suggests changes that could potentially make the use of probability in financial reports much more transparent.

> *"The modern life of an ordinary person is steeped in probabilistic concepts"*
> —Dr. Richard Durrett[1]

In order to resolve the existing negative reaction to the expanded use of probability calculations in financial performance reports, there needs to be a greater understanding of both the issues associated with practical applications and the ongoing scientific debate concerning the true meaning of probability.

This lack of understanding within the financial community is surprising given the pervasive requirement to use probability in many US financial standards. Probability calculations and assessments are required by the Financial Accounting Standards Board (FASB) under FAS Statement 133 (accounting for derivatives), FAS Statement 157 (fair value measurement), FIN 45 (guarantee valuation) and many other standards.

In my discussion of the conflicting scientific theories of probability, I will refer to an excellent treatise titled *Probability Is Symmetry*,[2] by Professor Krzysztof Burdzy, in which he defines the five laws of probability that govern: (1) the range of possible probabilities; (2) the disjointed nature of certain events; (3) the related concepts of independence and dependence; (4) the importance of symmetry; and (5) the related concepts of impossibility and inevitability.[3]

Perfectly Matched Transactions

Professor Burdzy's fifth law states that "… an event has probability [of] 1 if and only if it must occur." In order to make my discussion of probability in the marketing and trading environment as complete as possible, I will augment his fifth law to state that past events also have a probability of 1 (i.e., they "must have occurred"). As a starting point for this discussion, and using financial reporting terminology, all realized or past events should be considered to have a probability of 1.

Assuming that a value of "1" (or 100%) represents the most reliable information to preparers and users of financial statements, a valid follow-up question from preparers would be: *"Which future events also have a probability of 1 or close to 1?"*

In the context of marketing and trading activities, the short answer is that all forward fixed-price buy transactions that are perfectly "matched" with forward fixed-price sell transactions have a probability that is close to 1.

For example, if a company has a forward fixed-price contract with party "A" to buy crude oil during the month of March 2008 at $60 and also has a forward fixed-price contract with party "B" to sell crude oil in the month of March 2008 at $63, then there is a probability of close to 1 (or 100%) that the company will make $3 on this perfectly "matched" transaction — assuming that all other possible variables in the contracts are identical (same volume, same delivery point, same quality). This is also true for perfectly "matched" forward indexed-price contracts. These "matched" indexed-price transactions create a fixed margin that will not change regardless of future changes in the value of crude oil delivered during the month of March 2008.

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I would argue that perfectly "matched" forward-buy transactions and sell transactions, though unrealized, should therefore be considered almost equivalent to realized past transactions, in terms of the quality of their reported income — since both have probabilities of 1 or close to 1. The only difference between the former transactions (forward, matched and unrealized) and the latter transactions (past and realized) relates to performance/credit risk issues and to possible *force majeure* events (acts of God) that still might impact the future transactions but cannot change the past transactions.

Otherwise, these two classes of events, from market and price risk analysis perspectives, are almost indistinguishable in terms of their reliability and in terms of the quality of the earnings that they represent. Perfectly matched transactions are "time inviolate" in the sense that the calculated margin is fixed or frozen and will not change from the transaction date through the delivery date or at any other date in the future.

This class of "matched" transactions, however, is not distinguishable from less reliable "unmatched" transactions within the current financial standards, because all of the standards that require "fair value" measurements or assessments are "individual-transaction" specific — i.e., there is no differentiation made for the increase in probability associated with the "matching" process. More importantly, there is no concept of "matching" contained in any of the aforementioned financial standards, and yet these perfectly "matched" forward transactions are the most like realized past transactions with respect to their probability and level of confidence.

Practical Applications: Unmatched Indexed-Price Transactions and Unmatched Fixed-Price Transactions

As with perfectly matched future transactions, unmatched forward indexed-price contracts also have a high level of probability. Since a forward contract with an indexed price has a mark-to-market or fair value calculation of close to zero (assuming that the contract-indexed price is identical to the appropriate market-indexed price), then the probability that the fair value will equal zero from the transaction date through the future delivery date is close to 1 (100%).

Unlike past realized transactions, which have a probability of 1 — and also unlike perfectly matched forward transactions and unmatched indexed-price transactions, which both have a probability of close to 1 — the forward fixed-price contract has a changing probability (it is not "time inviolate"). The day on which the mark-to-market or fair value calculation is performed is the only day on which the probability of the expected profit or loss is close to 1 — and then only if there is immediate, ready liquidity available to close out that forward fixed price position.

At each day in the future, from the day after the date of the transaction through the delivery date, there will be a somewhat different mark-to-market or fair value calculation; this will become frozen only when the transaction is realized. Yet for purposes of financial performance reporting, a fair value calculation performed with respect to unmatched fixed-price transactions (which will change with time) is practically indistinguishable from the two previous classes of forward transactions (which are unchangeable with time or frozen).

This odd result exists because the new fair value hierarchy recently announced by FASB (in FAS 157) only distinguishes between the inputs used to perform the fair value calculations and does not distinguish between the different probabilities associated with these differing types of fair value calculations. The FASB fair value hierarchy is based on the various inputs used in performing a fair value measurement, which are described as follows:

- Level 1 fair value calculations are based on quoted prices in active markets for identical assets or liabilities;
- Level 2 fair value calculations are based on other significant observable [external] inputs; and
- Level 3 fair value calculations are based on significant unobservable [internal] inputs.

Earlier in this article, we offered examples of perfectly matched transactions examples, unmatched indexed-price transactions and unmatched fixed-price transactions. It is possible that all three of these forward transaction classes may fall into the same "level" of the fair value hierarchy for disclosure reporting purposes. But the first two types of transactions, though unrealized, have probabilities of close to 1 for all future periods (they are time inviolate or frozen), while the last type of transaction (i.e., the unmatched fixed price) will have a probability of close to 1 on the day of the calculation. Thereafter, that initial calculation of the fair value for an unmatched fixed price will have a much lower probability than 1 on all future days until the transaction is realized.

Practical Applications: Mark-to-Model Transactions

At the third level of the new fair value hierarchy (involving fair value calculations based on significant unobservable or internal inputs), we are almost always dealing with probabilities of less than 1. This is due to the fact that whenever we are performing FASB Level 3 calculations, by definition some of the external market indicators needed to perform the calculation are missing.

These are the cases in which a model of future prices or events must be built, typically using the concepts of extrapolation (continuing an existing price trend), interpolation (filling the gaps in a pricing curve) or correlation (estimating a forward curve in relation to the market pricing of a correlated asset or liability). In some more complicated cases, involving multiple variables and/or complex transactions, it becomes necessary to utilize Black-Scholes-type option models, Monte Carlo simulations, binomial tree calculations or other complex modelling techniques.

In all of these fair value cases, there is some level of uncertainty associated with the calculation. However, assuming that the resulting estimate of future pricing indicates a probability of greater than 50% (i.e., that it is more likely than not), then the fair value measurement itself is performed in relation to these modelled forward curves in the same manner that a calculation under FASB Levels 1 and 2 would be performed using external market inputs.

In other words, once the model is built, there is no explicit distinction made within the financial performance reports for the quantity or quality of profits calculated using external pricing indicators versus the quantity or quality of profits calculated using internal pricing models (other than the disclosure that a model has been employed in a Level 3 calculation).[4]

The "adjustments for risk" that are described in FAS 157 seem to refer to uncertainty as to performance, present value and other standard valuation questions — but not specifically to probability calculations and assessments used to develop pricing models. More importantly, the level of confidence or degree of probability associated with the pricing models used within these calculations is not visible either in the financial performance reports themselves or under the new additional disclosure requirements associated with FAS 157.

Practical Applications: Guarantee Valuation [5]

One area in which the use of probability in fair value calculations is more visible involves the valuation of guarantees under FIN 45. In these calculations, probability is not used to model one of the inputs to the fair value calculation (e.g., the forward curve), but rather probability is actually being used to perform the fair value calculation itself. Unfortunately, FIN 45 does not give much guidance on how to perform these probability calculations.

I propose that the use of probability within the fair value framework should be more carefully defined. In particular, FASB should adopt a formal process for calculating probability under FIN 45 and other financial standards. As a demonstration of this process with respect to FIN 45, a best in class "risk" approach should be based on two primary principles in guarantee valuation: (1) the value of a risk-free transaction is equal to the value of a risky transaction plus the value of the guarantee; and (2) the value of any contingent liability, including guarantees, equals its expected present value.

As defined by FASB,[6] the expected present value is the sum of the "probability-weighted" present values in a range of estimated cash flows, all discounted using the same interest rate convention. I also note that an optimum framework for valuing guarantees will employ FASB's fair value input hierarchy in the selection of both the valuation method and the inputs used to calculate the fair value.

Based on the requirements of FIN 45 (the two guarantee valuation principles stated above) and using best in class "risk" concepts, the three most logical methods for valuing guarantees using probability calculations can be described as follows:

METHOD ONE: MARKET VALUE METHOD

This method is the simplest to apply, but required market-based inputs are not always available. It is consistent with Level 1 of the fair value input hierarchy. Generally, the market value method can be applied in two cases: (1) If comparable risk-free and risky instruments exist with the liable party and the market values of these instruments are known. In this case, the value of the guarantee is simply the difference in the value of the risky and risk-free instruments. Or (2) if a fee is received for providing the guarantee. In this case, it is assumed that the guarantee's value is equal to the fee.

METHOD TWO: CREDIT SPREAD METHOD

This method is consistent with Level 2 of the fair value input hierarchy. It is based on the first valuation principle outlined earlier (i.e., "the value of a risk-free transaction is equal to the value of a risky transaction plus the value of the guarantee"). The value of a guarantee calculated this way, however, is valid only when the guarantor's probability of default is zero.

Nevertheless, we can approximate a guarantee's value when the guarantor is not default-free by assuming that the value of the guarantee is equal to the value of the guaranteed transaction less the value of the risky transaction.

The credit spread method can be used if

- the guarantee covers an obligation that is structured like a loan/bond;
- the credit spread of the liable party can be estimated; or
- the loss-given default (LGD) of the guarantee is the same as the LGD of the instruments used to imply the credit spread.

The credit spread is the difference in the risky rate and the rate with a guarantee. This is the method that is most widely used in valuing guarantees under FIN 45.

METHOD THREE: CONTINGENT CLAIMS VALUATION METHODS

Guarantee contracts represent contingent claims into the future. Consequently, the methodology for pricing contingent claims can be applied for estimating the value of guarantees. This valuation approach can be used to value almost any type of guarantee, including those that can be valued with the first two methods. It is consistent with Level 3 of the fair value input hierarchy, and it is based on the second principle ("the value of the guarantee is equal to the present value of the expected future guarantee payments").

Depending on how expectations are calculated — i.e., what probabilities are assigned to different events — different discount rates should be used. The following are the possible contingent claims valuation methods that can be used for valuing guarantees:

- Binomial tree with the actual probabilities of default.
- Risk-neutral (option-pricing) Valuation.
- Calculating the value of a loan guarantee explicitly as a put option.
- Binomial tree with underlying asset.
- Binomial tree with given risk-neutral probabilities of default.
- Monte Carlo simulation method.

Conflicting Theories of Probability

There are two major schools or theories of probability — the frequency theory and the subjective theory. Frequency theory (or the objective school) is generally consistent with classical statistics, while subjective theory is the basis for "Bayesian" statistics. (Symmetrical probability, described earlier in this article, is an attempt to reconcile these two major opposing schools of thought by asserting that all practical users of probability calculations act on the often unstated belief that probability is objectively accurate, whether or not the underlying theories are in agreement with the pragmatic behavior.)

In 1814, Pierre-Simon Laplace wrote:

The theory of chance consists in reducing all the events of the same kind to a certain number of cases equally possible — that is to say, to such as we may be equally undecided about in regard to their existence, and in determining the number of cases favorable to the event whose probability is sought. The ratio of this number to that of all the cases possible is the measure of this probability, which is thus simply a fraction whose numerator is the number of favorable cases and whose denominator is the number of all the cases possible.

This is a good description of classical statistics, which largely deals with "collectives" or large numbers of similar events. However, over the years, neither the frequency theory nor classical statistics have had much success with calculating the probability of individual events.

As a simple example, if we toss a true coin a large number of times, and 50% of the results are heads, we can then say with a high degree of confidence that during the next large number of coin tosses we are likely to see heads 50% of the time. But classical statistics has very little, if anything, to say about the likelihood of heads on the next individual coin toss.

In response to this and to other limitations of classical statistics, the subjective theory asserted that all probability estimates are subjective in nature. Followers of the subjective theory (e.g., the Reverend Thomas Bayes) assert that we should start with a "prior" set of data (whether large or small), and that from this "prior" data set we should build a logical and consistent model.

A test is then run of the model, which results in a "posterior" data set, and from this combination of the "prior" data set and the "posterior" data set, we can build a progressively better model. This iterative, Bayesian process is used throughout scientific research in searching for better medical treatments, improved materials, increased fuel efficiency, etc. It does not necessarily rely on a large number of prior events. The Bayesian process is also sometimes referred to as the assignment of "conditional" probability.

At its extreme case, followers of the subjective theory of probability assert that there is no such thing as objective "probability" and that all probability assessments are inherently subjective. Bruno de Finetti, the famous Italian probabilist and statistician, denied the existence of objective quantities representing probability.

Other theories of probability include the logical theory of probability (whose supporters include the economist John Maynard Keynes) and the propensity theory of probability. The logical theory is based on the "Principle of Indifference," which states that equal probabilities should be assigned to alternatives for which there is no known reason to be different. The propensity theory of probability is a version of the objective or frequency theory, and it asserts that "probability" is an objective property of things, just like other measurable physical phenomena (length, weight, etc.). Karl Raimund Popper, one of the most influential philosophers of science of the 20th century, was an early advocate of the propensity theory.

All of this demonstrates that there is ongoing uncertainty as to the true meaning of probability, especially when it comes to assigning probability to individual events (as opposed to "collectives" or large numbers of similar events). Yet it is these individual events that we are generally asked to assess by FASB in preparing financial performance reports.

"The first and most important step to take in addressing negative reactions to the use of probability calculations in financial performance reports is to make the process more transparent."

Given the fact that a large number of mathematicians will argue that all probability assessments are subjective in nature, is it any wonder that preparers and users of financial statements approach probability calculations and assessments with healthy skepticism?

Addressing Negative Reactions

The first and most important step to take in addressing negative reactions to the use of probability calculations in financial performance reports is to make the process more transparent. Under existing practice, we combine the changes in fair value sources of income with realized sources of income in a single income statement. By doing so, we ultimately confuse users of financial statements with respect to the quantity and quality of the hybrid reported income.

We also combine fair value measures of assets and liabilities with more traditional cost-based measures of assets and liabilities into a single balance sheet, and in so doing we confuse users of financial statements with respect to the size and quality of reported net equity.

In order to allow for a better understanding of the role of probability in the calculation of fair value, the first and best step would be to segregate all fair value measurements into a separate "Fair Value Statement."[7] In this statement, it would be appropriate not only to show the proposed FAS 157 hierarchy of inputs (see the three levels described on pg. 14), but also to show a "hierarchy of probability" that would carefully distinguish events that have probabilities of 1 or close to 1 from events that have lower probabilities (or that have changing probabilities over time).

> "In order to allow for a better understanding of the role of probability in the calculation of fair value, the first and best step would be to segregate all fair value measurements into a separate 'Fair Value Statement.'"

A second step that would help improve the confidence level in the quality of the information contained in financial statements would be to include some elements of the underlying mathematics in all financial standards that require the use of probability calculations and assessments. This could be done through the creation of a new FASB organization, perhaps organized along the lines of the existing Emerging Issues Task Force (EITF), that would review and address "risk"-related issues that arise in relation to existing and proposed financial standards.

This new group could be called the Fair Value Task Force (FVTF), and unlike the earlier "DIG" group that was organized only to interpret FAS 133, the FVTF should be more broadly charged with applying the best ideas in risk analysis and mathematics to existing and proposed financial standards that rely on the use of probability.

Finally, a recognition by FASB and IASB that their uses of terms like "probability" require careful consideration and study of the underlying mathematical theories and principles would go a long way toward solving the negative reactions that currently exist among preparers and users.

Given the extraordinary usefulness of many probability calculations, the required use of probability in new financial standards will only increase as time progresses. It is important that we solve the pragmatic concerns now and get the underlying mathematics right for the long haul. ■

FOOTNOTES:

1. Richard Durrett, "Current and Emerging Research Opportunities in Probability" (Cornell University, 2002). http://www.math.cornell.edu/~durrett/probrep/probrep.html.
2. Krzysztof Burdzy, *Probability Is Symmetry* (University of Washington, Seattle, 2003), http://www.math.washington.edu/~burdzy/Philosophy/book.pdf.
3. The Five Laws of Probability:
   (L1) Probabilities are numbers between 0 and 1, assigned to events whose outcome may be unknown.
   (L2) If events A and B cannot happen at the same time, the probability that one of them will occur is the sum of probabilities of the individual events, that is, P(A or B) = P(A) + P(B).
   (L3) If events A and B are physically unrelated then they are independent in the mathematical sense, that is, P(A and B) = P(A)P(B).
   (L4) If there exists a symmetry on the space of possible outcomes that maps an event A onto an event B, then the two events have equal probabilities, that is, P(A) = P(B).
   (L5) An event has a probability of 0 if and only if and only if it cannot occur. An event has a probability of 1 if and only if it must occur.
4. FAS 157 states that "… market participant assumptions include assumptions about risk, for example, the risk inherent in a particular valuation technique used to measure fair value (such as a pricing model) and/or the risk inherent in the inputs to the valuation technique. A fair value measurement should include an adjustment for risk if market participants would include one in pricing the related asset or liability, even if the adjustment is difficult to determine. Therefore, a measurement (for example, a "mark-to-model" measurement) that does not include an adjustment for risk would not represent a fair value measurement if market participants would include one in pricing the related asset or liability."
5. Special thanks to Georgi Vassilev, a PhD candidate in the Department of Economics at the University of Southern California, for his help with this section.
6. FASB, Statement of Financial Accounting Concepts No. 7, Using Cash Flow Information and Present Value in Accounting Measurements, February 2000.
7. For readers who are interested in this proposal, I refer them to my earlier article titled "Memo to FASB and IASB," *GARP Risk Review* (January/February 2005)

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